XII International Conference on Representation Theory and Workshop

15-24 August 2007, Toruń, Poland

ABSTRACTS

ICRA XII Workshop Abstracts

ICRA XII Workshop: August 15-18, 2007

PROGRAM

	Wednesday 15.08.2007	Thursday 16.08.2007	Friday 17.08.2007	Saturday 18.08.2007
$9^{00} - 10^{00}$	**S. König**	**S. König**	**S. König**	**B. Keller**
$10^{00} - 10^{30}$	*Coffee*	*Coffee*	*Coffee*	*Coffee*
$10^{30} - 11^{30}$	**B. Keller**	**B. Keller**	**M. Reineke**	**M. Reineke**
$11^{45} - 12^{45}$	**M. Reineke**	**S. Ariki**	**S. Ariki**	**S. Ariki**
$12^{45} - 15^{00}$	*Lunch*	*Lunch*	*Lunch*	*Lunch*
$15^{00} - 16^{00}$	**J.A. de la Peña**	**J.A. de la Peña**	**J.A. de la Peña**	
$16^{00} - 16^{30}$	*Coffee*	*Coffee*	*Coffee*	
$16^{30} - 17^{30}$	**O. Iyama**	**O. Iyama**	**O. Iyama**	
$17^{45} - 18^{45}$	**R.-O. Buchweitz**	**R.-O. Buchweitz**	**R.-O. Buchweitz**	

Susumu Ariki
Kyoto University, Kyoto, Japan

Finite dimensional Hecke algebras

Among finite dimensional algebras, Hecke algebras of finite type are a bit unique in the sense that the people working on the subject are different from those who usually come to ICRA. The theory has interactions with various fields of mathematics, as any fruitful examples do, and I will explain recent advances on the modular representation theory of the algebras in this minicource.

In the first lecture, after a brief review of Lusztig's theory, I explain results by Geck and his collaborators.

In the second lecture, I start with work by Dipper and James. Then, after a brief review of Kashiwara's theory of crystals, I explain Fock space theory for Hecke algebras of classical type.

In the third lecture, I will review other topics which are closely related to Hecke algebras. They include; rational Cherednik algebras and Rouquier's result on the quasi-hereditary cover, results related with Temperley-Lieb algebras and Stroppel's result.

Ragnar-Olaf Buchweitz
University of Toronto, Toronto, Canada

Quivers and algebraic geometry

Lecture 1: Categories derived from projective space

We will review and make explicit classical realisations of the bounded derived category of projective space and discuss related recent results and approaches, such as the monodromy of these realisations and their relation to Castelnuovo-Mumford regularity (Costa & Miro-Roig), extensions to other rational varieties (King) or to projective (Orlov) and some affine (Segal) bundles over them.

Lecture 2: Projective geometry versus maximal Cohen-Macaulay modules

We will present and discuss Orlov's result that compares, explicitly, the derived category of coherent sheaves on the underlying projective "space" of a not necessarily commutative graded Gorenstein algebra with the triangulated stable category of graded Maximal Cohen-Macaulay (MCM) modules over that algebra. It yields a complete "classification" of MCMs through projective data and a finite quiver, and conversely, say, for hypersurfaces, describes the projective geometry through matrix factorisations. We will outline further work, in progress, for more general toric varieties that comes from Physics (Herbst, Hori, Page et al.) and indicate the string-theoretic input via super-potentials.

Lecture 3: Noncommutative desingularisations of varieties of minors and maximal Cohen-Macaulay modules of homomorphisms

Here we present work, still in progress, with Graham Leuschke and Michel van den Bergh, exhibiting the desingularisations in the title and showing how these results address basic questions from Commutative, if not Linear Algebra. They relate to the material from the previous lectures in various ways, perhaps most importantly via representations of graded algebras supported on "admissible windows", a notion motivated, yet again, by String Theory.

Osamu Iyama
Nagoya University, Nagoya, Japan

Auslander-Reiten theory revisited

(joint with: *I. Reiten, Y. Yoshino, A. Buan, J. Scott, I. Burban and B. Keller*)

We shall discuss Auslander-Reiten theory and its higher analogue. This is also related to cluster categories, cluster algebras, Calabi-Yau algebras, non-commutative crepant resolutions of singularities and representation dimension.

Let $\mathcal{X}$ be an exact category and $n \geq 1$ an integer. We call an object $M \in \mathcal{X}$ *n-cluster tilting* if

$$\begin{aligned} \mathrm{add}M &= \{X \in \mathcal{X} \mid \mathrm{Ext}^i(M,X) = 0 \ (0 < i < n)\} \\ &= \{X \in \mathcal{X} \mid \mathrm{Ext}^i(X,M) = 0 \ (0 < i < n)\}. \end{aligned}$$

Obviously 1-cluster tilting objects are additive generators, and 2-cluster tilting objects are usually called cluster tilting.

For an n-cluster tilting object $M \in \mathrm{mod}\Lambda$, the category $\mathcal{M} := \mathrm{add}M$ has

(a) an equivalence $\tau_n : \underline{\mathcal{M}} \xrightarrow{\sim} \overline{\mathcal{M}}$ (*n-AR translation*),

(b) a functorial isomorphism $\underline{\mathrm{Hom}}_\Lambda(X,Y) \simeq D\mathrm{Ext}^n_\Lambda(X, \tau_n Y)$ for any $X, Y \in \mathcal{M}$ (*n-AR duality*),

(c) for any indecomposable non-projective $X \in \mathcal{M}$, an exact sequence

$$0 \to \tau_n X \xrightarrow{b} M_{n-1} \to \cdots \to M_0 \xrightarrow{a} X \to 0 \quad (M_i \in \mathcal{M})$$

with a minimal right almost split map a and a minimal left almost split map b in $\mathcal{M}$ (*n-almost split sequence*),

(d) an endomorphism algebra $\Gamma := \mathrm{End}_\Lambda(M)$ satisfying

$$\mathrm{gl.dim}\Gamma \leq n+1 \leq \mathrm{dom.dim}\Gamma$$

(*Auslander algebra of global dimension* $n+1$).

We observe these general results through the following examples.

(1) For any representation-finite hereditary algebra Λ_1, we have the following inductive construction of algebras with n-cluster tilting objects.

For any $n \geq 1$, there exists an algebra Λ_n with a tilting Λ_n-module T_n such that the category $T_n^\perp := \{X \in \mathrm{mod}\Lambda_n \mid \mathrm{Ext}^i_{\Lambda_n}(T_n, X) = 0 \ (i > 0)\}$ contains an n-cluster tilting object M_n satisfying $\Lambda_{n+1} = \mathrm{End}_{\Lambda_n}(M_n)$.

The set of vertices in the quiver of Λ_{n+1} has a similar structure with integer points $\mathbf{Z}^{n+1}$ in the $(n+1)$-dimensional Euclidean space $\mathbf{R}^{n+1}$.

(2) Let Λ be a preprojective algebra of non-Dynkin type and W the corresponding Coxeter group. Then there exists a bijection $w \mapsto I_w$ between elements w in W and a two-sided ideal I of Λ which is a tilting Λ-module and Λ/I is artinian. For any reduced expression of w, we have a 2-cluster tilting object in the category $\mathrm{Sub}(\Lambda/I_w)$ (cf. recent work of Geiss-Leclerc-Schröer).

(3) Let k be an algebraically closed field of characteristic zero, G a finite subgroup of $\mathrm{GL}_n(k)$, $S = k[[x_1, \cdots, x_d]]$ a formal power series ring and $\Lambda := S^G$ the invariant subring. If Λ is an isolated singularity, then the category $\mathrm{CM}(\Lambda)$ contains a $(d+1)$-cluster tilting object S.

(4) Let k be in (3) and $\Lambda := k[[x, y]]/(f)$ $(f \in (x, y))$ a one-dimensional reduced hypersurface singularity. Then the category $\mathrm{CM}(\Lambda)$ has a 2-cluster tilting object if and only if the factorization $f = f_1 \cdots f_n$ of f satisfies $f_i \notin (x, y)^2$ $(1 \leq i \leq n)$. In this case all cluster tilting objects in $\mathrm{CM}(\Lambda)$ are indexed by elements in the symmetric group of degree n. A key step is a comparison of mutation of cluster tilting objects with that of tilting modules.

Bernhard Keller
Université Paris 7, Paris, France

Calabi-Yau triangulated categories

A triangulated category whose morphism spaces are finite-dimensional over a ground field k is d-Calabi-Yau if the dth power of its suspension functor is a Serre functor. This holds iff it admits Auslander-Reiten sequences and the Auslander-Reiten translation is isomorphic to the $(d-1)$th power of the suspension functor. The definition is due to Kontsevich and the terminology inspired by the example of the derived category of coherent sheaves on a d-dimensional Calabi-Yau variety. Calabi-Yau categories have recently attracted the attention of representation theorists because they appear in the study of periodicity phenomena and in the representation theoretic approach to cluster algebras. In these lectures, we will report on some aspects of these developments.

Steffen König
Universität Köln, Köln, Germany

Diagram algebras

The term diagram algebras refers to classes of algebras that share common features with group algebras of symmetric groups. These include Hecke algebras of type A, Temperley-Lieb algebras, Brauer algebras, Birman-Murakami-Wenzl algebras, partition algebras, semigroup algebras of rook monoids, affine Temperley-Lieb algebras, affine Hecke algebras, and others. Such algebras (of finite or of infinite dimension) have come up in invariant theory, in knot theory, in statistical mechanics, in combinatorics, in Lie theory, in number theory and in various other contexts, often as endomorphism algebras of certain representations.

In these lectures I will introduce some of these examples and discuss various features of their ring structure and of their representation theory, focussing on topics that may be of interest to people working in representation theory of finite dimensional algebras, by exhibiting new phenomena, by suggesting axiomatic definitions and by posing challenges.

- Classifications: generically semisimple and generally wild?
- Homological properties: derived categories, finitistic dimension, stratifications - finding structure in combinatorial situations (sometimes).
- Axiomatics by combinatorial or by homological properties: the intersection of cellular and stratified is obvious, is it?
- The Hemmer-Nakano phenomenon: symmetric and finite global dimension are disjoint situations, are they? Some almost relatively true statements.
- Affine situations: can we handle infinite dimensional algebras?
- Bases and their combinatorics and statistics: why should one care? Is there a connection to public transport in Cuernavaca?

These lectures will touch upon the work of many authors. In particular, I will discuss recent and current joint work with Changchang Xi and with Robert Hartmann, Anne Henke and Rowena Paget.

José Antonio de la Peña

Universidad Nacional Autónoma de México, México City, México

Tame algebras and quadratic forms

Let k denote an algebraically closed field and A a finite dimensional k-algebra. We consider algebras A presented by a quiver with relations in the form $A = kQ/I$, where Q has no oriented cycles. Then the *Tits quadratic form* $q_A\colon \mathbb{Z}^{Q_0} \to \mathbb{Z}$ is introduced by

$$q_A(v) = \sum_{i\in Q_0} v(i)^2 - \sum_{(i\to j)\in Q_1} v(i)v(j) + \sum_{i,j\in Q_0} r(i,j)v(i)v(j)$$

where Q_0 (resp. Q_1) denotes the set of vertices (resp. arrows) of Q and $r(i,j)$ is the number of elements in $R\cap I(i,j)$ whenever R is a minimal set of generators of I contained in $\bigcup_{i,j\in Q_0} I(i,j)$. This quadratic form was introduced by Bongartz [B] as generalization of considerations by Tits, Gabriel and Bernstein-Gelfand-Ponomarev in the study of the representations of hereditary algebras $A = kQ$ of finite representation type.The purpose of our lectures is to provide an overview on the concepts and techniques leading to the recent proof of the following:

Theorem ([BruPS]). *Let A be a strongly simply connected algebra. Then A is tame if and only if the Tits form q_A of A is weakly non-negative (that is, $q_A(v) \geq 0$ for any $v \in \mathbb{N}^{Q_0}$).*

This Theorem is a natural generalization of ealier results on path algebras $A = kQ$ [G], [N], representation-finite algebras [B], observations by Brenner [Br], and several particular cases proved by several authors along the years. As an important consequence of the Theorem we get that every wild strongly simply connected algebra is strictly wild. Moreover, any such an algebra has a convex subcategory with at most 10 objects which is wild.

In Lecture 1 we shall recall the notion of tameness and show that a tame algebra A has q_A weakly non-negative [P]. We show that the converse holds for certain classes of algebras (tilted algebras, coil enlargements of tame concealed algebras, among others) but not for arbitrary triangular algebras.

In Lecture 2 we recall definitions and examples of simply connected and strongly simply connected algebras [S1]. We present the main results of the representation theory of strongly simply connected algebras of polynomial growth [AS], [PS1], [PS2], [S2].

In Lecture 3 we introduce the concept of a $\mathbb{D}$-algebra (a class of tame algebras whose representation theory is controlled by a deformation of a special biserial algebra). We present results on minimal non-polynomial growth strongly simply connected algebras [NoS], some reduction considerations [PS3] and geometric arguments [Ge] which imply the main result.

Main references

[AS] I. Assem and A. Skowroński: Coil and multicoil algebras. In: Representation Theory of Algebras and Related Topics, CMS Conf. Proc. 14 (1993), 29-68.

[AST] I. Assem, A. Skowroński and B. Tomé: Coil enlargements of algebras, Tsukuba J. Math. 19 (1995), 453-479.

[B] K. Bongartz: Algebras and quadratic forms, J. London Math. Soc. 28 (1984), 1-12.

[Br] S. Brenner: Quivers with commutativity conditions and some phenomenology of forms. In: Representations of Algebras, Springer LNM 488 (1975), 29-53.

[BruPS] T. Brüstle, J.A. de la Peña and A. Skowroński: Tame algebras and the Tits quadratic form. Preprint.

[G] P. Gabriel: Unzerlegbare Darstellungen I, Manuscripta Math. 6 (1972), 71-103.

[Ge] C. Geiss: On degenarations of tame and wild algebras. Arch. Math. 64 (1995), 11-16.

[N] L.A. Nazarova: Representations of quivers of infinite type. Izv. Akad. Nauk SSSR 37 (1973), 752-791.

[NoS] R. Nörenberg and A. Skowroński: tame minimal non-polynomial growth simply connected algebras. Colloq. Math. 73 (1997), 301-330.

[P] J.A. de la Peña: On the representation type of one-point extensions of tame concealed algebras. Manuscripta Math. 61 (1988), 183-194.

[PS1] J.A. de la Peña and A. Skowroński: Geometric and homological characterizations of polynomial growth strongly simply connected algebras. Inventiones Math. 126 (1996), 287-296.

[PS2] J.A. de la Peña and A. Skowroński: The Tits and Euler forms of a tame algebra, Math. Ann. 315 (1999), 37-59.

[PS3] J.A. de la Peña and A. Skowroński: Substructures of algebras with weakly non-negative Tits form. Extracta Math. to appear (2007).

[S1] A. Skowroński: Simply connected algebras and Hochschild cohomologies. In: Representation of Algebras, CMS Conf. Proc. 14 (1993), 431-447.

[S2] A. Skowroński: Simply connected algebras of polynomial growth. Compositio Math. 109 (1997), 99-133.

Markus Reineke
Bergische Universität Wuppertal, Wuppertal, Germany

Moduli of representations of quivers

The talks will survey the construction of moduli spaces of representations of quivers, their geometric properties, and their connection to classification problems. More precisely, the following aspects will be addressed:

The role of (semi-)stable representations will be discussed. The construction of moduli spaces will be reviewed. The relevance of a topological and arithmetic study of the moduli spaces for classification problems in the representation theory of wild quivers will be discussed. Several results on geometric invariants of the moduli spaces will be reviewed. The role of Hall algebras for the study of moduli spaces will be explained. Variants of the original moduli spaces, motivated by geometric problems, will be constructed. Emphasis will be placed on concrete examples, open problems and future perspectives.

ICRA XII Conference Abstracts

ICRA XII Conference: August 20-24, 2007

PROGRAM

	Monday 20.08.2007	Tuesday 21.08.2007	Wednesday 22.08.2007	Thursday 23.08.2007	Friday 24.08.2007
$9^{00} - 9^{50}$	**I. Reiten**	**B. Leclerc**	**C. Riedtmann**	plenary	plenary
$10^{05} - 10^{35}$	plenary	plenary	plenary	plenary	plenary
$10^{35} - 11^{05}$	*Coffee*	*Coffee*	*Coffee*	*Coffee*	*Coffee*
$11^{05} - 11^{35}$	plenary	plenary	plenary	plenary	plenary
$11^{50} - 12^{40}$	**K. Erdmann**	**A. Zelevinsky**	**H. Derksen**	plenary	plenary
$12^{40} - 14^{40}$	*Lunch*	*Lunch*	*Lunch*	*Lunch*	*Lunch*
$14^{40} - 15^{00}$	parallel sessions	parallel sessions	*free*	parallel sessions	parallel sessions
$15^{10} - 15^{30}$	parallel sessions	parallel sessions		parallel sessions	parallel sessions
$15^{40} - 16^{00}$	parallel sessions	parallel sessions		parallel sessions	parallel sessions
$16^{00} - 16^{30}$	*Coffee*	*Coffee*		*Coffee*	*Coffee*
$16^{30} - 16^{50}$	parallel sessions	parallel sessions		parallel sessions	parallel sessions
$17^{00} - 17^{20}$	parallel sessions	parallel sessions		parallel sessions	parallel sessions
$17^{30} - 17^{50}$	parallel sessions	parallel sessions		parallel sessions	parallel sessions
$18^{05} - 18^{55}$	**C.M. Ringel**	**W. Crawley-Boevey**		**H. Lenzing**	**J. Xiao**

1 William Crawley-Boevey

University of Leeds, Leeds, United Kingdom

Remarks on the Deligne-Simpson problem

I shall discuss some aspects of my work on the Deligne-Simpson problem, and some open problems.

2 Harm Derksen

University of Michigan, Ann Arbor, USA

Combinatorics of quiver representations

(joint with: *Calin Chindris and Jerzy Weyman*)

We analyse stability for quiver representations. This leads to the notion of Schur sequences which is a natural generalization of exceptional sequences. There are many applications to combinatorics of Littlewood-Richardson coefficients. We extend the description of Knutson-Tao-Woodward of the walls of the Klyachko cone to a description of all faces of the Klyachko cone. Also, with our methods we construct a counterexample to Okounkov's log-concavity conjecture for LR-coefficients.

3 Karin Erdmann

University of Oxford, Oxford, United Kingdom

Deformed mesh algebras of generalized Dynkin type

(joint with: *Jerzy Białkowski and Andrzej Skowroński*)

In [BES] we give a homological characterisation of a class of algebras with periodic bimodule resolution, which contains all preprojective algebras of Dynkin type ADE. This generalizes naturally, and gives also rise to algebras associated to all other (non-simply laced) Dynkin types. We introduce the class of deformed mesh algebras of generalized Dynkin types $\mathbb{A}_n(n \geq 1)$, $\mathbb{B}_n, (n \geq 2)$, $\mathbb{C}_n(n \geq 3)$, $\mathbb{D}_n(n \geq 4)$, $\mathbb{E}_6$, $\mathbb{E}_7$, $\mathbb{E}_8$, $\mathbb{F}_4$, $\mathbb{G}_2$, $\mathbb{L}_n(n \geq 1)$. These are finite dimensional self-injective algebras over an algebraically closed field K with periodic bimodule resolutions. The class of these algebras contains the stable Auslander algebras of hypersurface singularities of finite Cohen-Macaulay type, and it also contains the basic connected finite dimensional self-injective algebras whose stable module categories are 2-Calabi-Yau.

We show that the class of deformed mesh algebras of generalized Dynkin types coincides with the class of all basic connected finite dimensional self-injective algebras for which the third syzygy $\Omega^3 S$ of any non-projective simple module S is simple.

[BES] J. Białkowski, K. Erdmann, A. Skowroński, *Deformed preprojective algebras of generalized Dynkin type.* Trans. A.M.S. 359(2007) 2625-2650.

4 Bernard Leclerc

Université de Caen, Caen, France

Subcategories of modules over preprojective algebras and cluster structures

(joint with: *Christof Geiss and Jan Schröer*)

Let G be a simple algebraic group of type A, D, E, and N a maximal unipotent subgroup. We have shown that the study of rigid modules over the preprojective algebra Λ attached to G leads to a representation-theoretic interpretation of the cluster algebra structure on the ring $\mathbb{C}[N]$ of polynomial functions on N. In order to obtain similar interpretations for the cluster algebras attached to Schubert cells, to Grassmannians, and to introduce as well some new cluster algebras for general partial flag varieties G/P, we are lead to replace the category $\mathrm{mod}(\Lambda)$ by some appropriate Frobenius subcategories and to study their rigid modules. I will survey these results and, if time allows, I will also discuss some generalizations to the Kac-Moody case.

5 Helmut Lenzing

Universität Paderborn, Paderborn, Germany

Extended canonical algebras and Fuchsian singularities

(joint with: *José Antonio de la Peña*)

The talk deals with the shape of the bounded derived category $D(B) = D^b(\mathrm{mod}(B))$ of finite dimensional modules over an extended canonical algebra B. Such an algebra arises as the one-point extension of a canonical algebra A by an indecomposable projective (or injective) module and always has global dimension two. For the problem under investigation it does not matter which indecomposable projective (or injective) is taken as the extension-module, that is, $D(B)$ is an invariant of the situation. Surprisingly, the shape of $D(B)$ depends sensibly on the representation type of the canonical algebra A:

(1) If A is tame domestic, then $D(B)$ is the bounded derived category of representations of a wild quiver, whose underlying graph is doubly extended Dynkin.

(2) If A is tame tubular of type $(p_1, \ldots, p_t)$, with the weights p_i in ascending order, then $D(B)$ is equivalent to the bounded derived category of representations of a wild canonical algebra of type $(p_1, \ldots, p_{t-1}, p_t + 1)$.

(3) If A is wild, then $D(B)$ is neither derived-equivalent to a hereditary nor a canonical algebra. Instead it is equivalent to the triangulated category

$$D_{Sg}^{\mathbb{Z}}(R) = D(\mathrm{mod}^{\mathbb{Z}}(R))/D^b(\mathrm{mod}^{\mathbb{Z}}(R))$$

of the graded surface singularity $R = \bigoplus_{n\geq 0} \mathrm{Hom}(L, \tau^n L)$ associated to A (or to the weighted projective line $\mathbb{X}$ associated to A). Here, L denotes a line bundle on $\mathbb{X}$ and τ the Auslander-Reiten translation in the category of coherent sheaves on $\mathbb{X}$. Following Buchweitz (1987) the stable category $\underline{\mathrm{CM}}^{\mathbb{Z}}(R)$ of graded (maximal) Cohen-Macaulay modules over R provides an alternative description of $D(B)$. [The singularities R are called Fuchsian, since for the base field of complex numbers they are graded rings of automorphic forms with respect to the action of a Fuchsian group on the upper complex half plane.]

In all three cases, a spectral analysis of the extended canonical algebra yields additional interesting insight.

Our results are related to recent work by Orlov (2005), Ueda (2006) and Saito-Takahashi (unpublished).

6 Idun Reiten

Norwegian University of Science and Technology, Trondheim, Norway

Construction of 2-Calabi-Yau categories from preprojective algebras

(joint with: *Aslak Bakke Buan, Osamu Iyama and Jeanne Scott*)

We use tilting modules over preprojective algebras A of non Dynkin quivers Q to construct finite dimensional factor algebras B of A which are Gorenstein of dimension at most 1, and where the stable category C of SubB is 2-Calabi-Yau. The factor algebras B are associated with elements w in the Coxeter group of Q, and we describe cluster tilting objects T in SubB and in C, associated with different reduced expressions of w. We also describe the quivers of the endomorphism algebras of the cluster tilting objects T in SubB or in C directly from the reduced expressions of w. The class of 2-Calabi-Yau categories we construct this way contains the cluster categories and the stable categories for preprojective algebras of Dynkin type. We work over an algebraically closed field k. Some results have been obtained independently by Geiss, Leclerc and Schroer with completely different methods.

7 Christine Riedtmann

University of Berne, Berne, Switzerland

Semi-invariants for quiver representations and their zero sets

Let k be an algebraically closed field with $char k = 0$ and $Q = (Q_0, Q_1, s, t)$ a finite quiver with vertex set $Q_0 = \{1, \ldots, n\}$, where $s\alpha$ and $t\alpha$ denote the starting and the terminating vertex of $\alpha \in Q_1$, respectively. For a quiver Q and a vector $\mathbf{d} = (d_1, \ldots, d_n) \in \mathbb{N}^n$, the set $\mathrm{rep}(Q, \mathbf{d}) = \prod_{\alpha \in Q_1} \mathrm{Hom}(k^{d_{s\alpha}}, k^{d_{t\alpha}})$, "the space of representations of Q with dimension vector $\mathbf{d}$", is a vector space on which the group $\mathrm{Gl}(\mathbf{d}) = \prod_{i=1}^n \mathrm{Gl}(d_i)$ and its subgroup $\mathrm{Sl}(\mathbf{d}) = \prod_{i=1}^n \mathrm{Sl}(d_i)$ act by $(g \star X)(\alpha) = g_{t\alpha} X(\alpha) g_{s\alpha}^{-1}$ for $g = (g_1, \ldots, g_n) \in \mathrm{Gl}(\mathbf{d}), X \in \mathrm{rep}(Q, \mathbf{d})$, and $\alpha \in Q_1$.

A polynomial function $f \in k[\mathrm{rep}(Q, \mathbf{d}]$ is called an invariant on $\mathrm{rep}(Q, \mathbf{d})$ under $\mathrm{Gl}(\mathbf{d})$ or $\mathrm{Sl}(\mathbf{d})$ if f is constant on $\mathrm{Gl}(\mathbf{d})$- or $\mathrm{Sl}(\mathbf{d})$-orbits, respectively. Generators for the vector space of $\mathrm{Gl}(\mathbf{d})$-invariants have been given in [1]; if Q does not contain oriented cycles, any $\mathrm{Gl}(\mathbf{d})$-invariant on $\mathrm{rep}(Q, \mathbf{d})$ is constant. In many cases there are non-constant $\mathrm{Sl}(\mathbf{d})$-invariants, however. *From now on we assume that Q does not contain oriented cycles.*

As $\mathrm{Sl}(\mathbf{d})$ is a normal subgroup of $\mathrm{Gl}(\mathbf{d})$, the torus $H = \mathrm{Gl}(\mathbf{d}) / \mathrm{Sl}(\mathbf{d})$ acts on the space $k[\mathrm{rep}(Q, \mathbf{d})]^{\mathrm{Sl}(\mathbf{d})}$ of $\mathrm{Sl}(\mathbf{d})$-invariants, inducing a weight space decomposition $k[\mathrm{rep}(Q, \mathbf{d})]^{\mathrm{Sl}(\mathbf{d})} = \oplus_{\chi \in X(H)} \mathrm{SI}(Q, \mathbf{d})_\chi$, where χ ranges over all characters on H, i.e. all polynomial functions on H which are group homomorphisms from H to k^*. Here $\mathrm{Sl}(Q, \mathbf{d})_\chi$, the space of all semi-invariants of weight χ, is the vector space of all $\mathrm{Sl}(\mathbf{d})$-invariants f for which $(g \star f)X = \chi(g)f(g^{-1} \star X)$ for all X and all g. A character χ on H is necessarily of the form $\chi(g_1, \ldots, g_n) = (det g_1)^{\chi_1} \cdots (det g_n)^{\chi_n}$ for some integers $\chi_1, \ldots, \chi_n$, and therefore the character group $X(H)$ can be identified with $\mathbb{Z}^n \simeq \mathrm{Hom}(\mathbb{Z}^n, \mathbb{Z})$.

For $\mathbf{d}, \mathbf{e} \in \mathbb{N}^n$, set $\langle \mathbf{d}, \mathbf{e} \rangle = \sum_{i=1}^n d_i e_i - \sum_{\alpha \in Q_1} d_{s\alpha} e_{t\alpha}$. If $X \in \mathrm{rep}(Q, \mathbf{d}), Y \in \mathrm{rep}(Q, \mathbf{e})$ the linear map

$$F_{X,Y} : \prod_{i=1}^{n} \mathrm{Hom}_k(k^{d_i}, k^{e_i}) \to \prod_{\alpha \in Q_1} \mathrm{Hom}_k(k^{d_{s\alpha}}, k^{d_{t\alpha}})$$

defined by $F_{X,Y}(f_1, \ldots, f_n) = (f_{t\alpha} X(\alpha) - Y(\alpha) f_{s\alpha}; \alpha \in Q_1)$ is given by a square matrix (with respect to some bases) provided $\langle \mathbf{d}, \mathbf{e} \rangle = 0$. By [4], $c_Y = \det F_{X,Y}$, which is defined uniquely up to multiplication by a non-zero scalar, is a semi-invariant on $\mathrm{rep}(Q, \mathbf{d})$ of weight $-\langle -, \mathbf{e} \rangle$, and these invariants c_Y generate the vector space of $\mathrm{Sl}(\mathbf{d})$-invariants by [2]. Note that the kernel of $F_{X,Y}$ is just $\mathrm{Hom}_Q(X, Y)$, and thus $c_Y \neq 0$ iff $\mathrm{Hom}_Q(X, Y) = 0$ generically on $\mathrm{rep}(Q, \mathbf{d})$.

By Hilbert's theorem, $k[\mathrm{rep}(Q, \mathbf{d}]^{\mathrm{Sl}(\mathbf{d})}$ is a finitely generated algebra; generators for and the structure of this algebra are known in a few cases only, in particular if the action of $\mathrm{Gl}(\mathbf{d})$ is prehomogeneous, i.e. if $\mathrm{rep}(Q, \mathbf{d})$ contains an open (dense) $\mathrm{Gl}(\mathbf{d})$-orbit. In that case, if $f_1, \ldots, f_s$ are irreducible polynomials whose zero sets are the irreducible components with codimension 1 of the complement of the open orbit, the polynomials $f_1, \ldots, f_s$ generate the algebra $k[\mathrm{rep}(Q, \mathbf{d})]^{\mathrm{Sl}(\mathbf{d})}$, and they are algebraically independent [4].

The set $\mathcal{Z}_{Q,\mathbf{d}}$ of common zeros of all semi-invariants on $\mathrm{rep}(Q, \mathbf{d})$ which vanish at 0 contains algebraic information: If it is a regular variety and a complete intersection, which means that it is the zero set of an ideal generated by just as many polynomials as its codimension indicates, then the algebra $k[\mathrm{rep}(Q, \mathbf{d})]$ is a free module over $k[\mathrm{rep}(Q, \mathbf{d})]^{\mathrm{Sl}(\mathbf{d})}$ [5]. If the action is prehomogeneous, these conditions are always satisfied for multiples $N\mathbf{d}$ of the given dimension vector $\mathbf{d}$ with $N \geq N_0$ for a natural number N_0 depending on Q [3].

Most of these results are true for algebraically closed fields of arbitrary characteristic; some carry over to certain varieties of representations of quivers with relations.

References

[1] L. Le Bruyn and C. Procesi, Semisimple representations of quivers, *Trans. Amer. Math. Soc.* **317** (1990), 585–598.

[2] H. Derksen and J. Weyman, Semi-invariants of quivers and saturation for Littlewood-Richardson coefficients, *J. Amer. Math. Soc.* **13** (2000), 467–479.

[3] Ch. Riedtmann and G. Zwara, On the zero set of semi-invariants for quivers, *Ann. Sci. Ecole Norm. Sup.* **36** (2003), 969–976.

[4] A. Schofield, Semi-invariants of quivers, *J. London Math. Soc.* **43** (1991), 385–395.

[5] G. W. Schwarz, Lifting smooth homotopies of orbit spaces, *Inst. Hautes Etudes Sci. Publ. Math.* **51** (1980), 37–135.

8 Claus Michael Ringel

Universität Bielefeld, Bielefeld, Germany

The relevance and the ubiquity of Prüfer modules

Let R be a ring. An R-module M will be said to be a Prüfer module provided there exists a locally nilpotent and surjective endomorphism of M with kernel of finite length. In the lecture we want to outline the relevance, but also the ubiquity of Prüfer modules. Any Prüfer module which is not of finite type gives rise to a generic module, thus to infinite families of indecomposable modules with fixed endo-length (here we are in the setting of the second Brauer-Thrall conjecture). The aim of the lecture is to exhibit a construction procedure for Prüfer modules which yields a wealth of such modules.

9 Jie Xiao

Tsinghua University, Beijing, China

Green's formula with C*-action and Caldero-Keller's formula for cluster algebras

(joint with: *Fan Xu*)

It is known that Green's formula over finite fields gives rise to the comultiplications of Ringel-Hall algebras and quantum groups (see [Green], also see [Lusztig]). In this talk, we deduce the projective version of Green's formula in a geometric way. Then following the method of Hubery in [Hubery2005], we apply this formula to prove Caldero-Keller's multiplication formula for acyclic cluster algebras of arbitrary type. The talk is based on a joint work with F.Xu.

10 Andrei Zelevinsky

Northeastern University, Boston, USA

Mutations for quivers with potentials

(joint with: *Harm Derksen and Jerzy Weyman*)

We study quivers with relations of a special kind given by non-commutative analogs of Jacobian ideals in the completed path algebra. They provide a representation-theoretic interpretation of quiver mutations at arbitrary vertices, thus giving a far-reaching generalization of classical Bernstein-Gelfand-Ponomarev reflection functors. Motivations for this work come from several sources: superpotentials in physics, Calabi-Yau algebras, cluster algebras.

11 István Ágoston

Eötvös University, Budapest, Hungary

Approximations of algebras by standardly stratified algebras

(joint with: *Vlastimil Dlab and Erzsébet Lukács*)

We define two operators on the class of all finite dimensional algebras.

Generalizing an earlier construction by Dlab and Ringel, we show that for a given algebra A there is a unique basic algebra, denoted by $\Sigma(A)$, such that $\Sigma(A)$ is standardly stratified (i.e. the right regular module $\Sigma(A)_{\Sigma A}$ is filtered by so-called standard modules $\Delta(i)$), and the subcategories of modules with standard filtration over A and over $\Sigma(A)$ are equivalent. A parallel result states the existence of a unique basic algebra $\Omega(A)$ such that the opposite algebra of $\Omega(A)$ is standardly stratified (this is equivalent to the requirement that the right regular module $\Omega(A)_{\Omega(A)}$ is filtered by so-called proper standard modules $\overline{\Delta}(i)$) and the subcategories of modules with proper standard filtration over A and over $\Omega(A)$ are equivalent.

We show that – apart from a few easy and natural exceptions – the preimages of both operators contain infinitely many non-equivalent algebras; thus in most cases infinitely many algebras have equivalent subcategories of modules with standard or proper standard filtration, respectively.

We also show that the repeated application of the operators Σ and Ω stabilizes after a finite number of steps in a so-called properly stratified algebra (these are algebras for which both the right and the left regular module is filtered by standard modules). Namely, if n denotes the number of isomorphism types of simple A-modules, then $(\Omega\Sigma)^{n-1}(A)$ is properly stratified, hence it is fixed under the action of both operators. Thus if we consider the Cayley graph of the semigroup action of the operators Σ and Ω restricted to the class of algebras which are standardly stratified on either side, the components of this graph are trees and these components are parametrized by properly stratified algebras.

12 Claire Amiot

Université Paris 7, Paris, France

On the structure of triangulated categories with finitely many indecomposables

In this talk, we study the problem of classifying triangulated categories with finite-dimensional morphism spaces and finitely many indecomposables over an algebraically closed field. We obtain a new proof of the following result due to Xiao and Zhu: the Auslander-Reiten quiver of such a category is of the form $\mathbb{Z}\Delta/G$ where Δ is a disjoint union of simply laced Dynkin diagrams and G a weakly admissible group of automorphisms of $\mathbb{Z}\Delta$. Then we prove that for 'most' groups G, the category $\mathcal{T}$ is standard, *i.e.* k-linearly equivalent to an orbit category $\mathcal{D}^b(\mathsf{mod}k\Delta)/\Phi$. This happens in particular when $\mathcal{T}$ is maximal d-Calabi-Yau with $d \geq 2$. Moreover, if $\mathcal{T}$ is standard and algebraic, we can even construct a triangle equivalence between $\mathcal{T}$ and the corresponding orbit category.

13 Lidia Angeleri Hügel

Università dell'Insubria, Varese, Italy

Tilting modules arising from ring epimorphisms

(joint with: *Javier Sánchez*)

Given a ring R, we investigate tilting modules of the form $S \oplus S/R$ for certain injective ring epimorphisms $R \to S$. In particular, we are interested in tilting modules arising from Schofield's universal localization. For some rings, in this way one obtains a classification of all tilting modules.

14 Javad Asadollahi Dehaghi

Shahre-Kord University, Shahre-Kord, Iran

Complete cohomology for complexes

In this talk, we introduce and study a complete cohomology theory for complexes, which provides an extended version of Tate-Vogel cohomology in the setting of arbitrary complexes over associative rings.

15 Hideto Asashiba

Shizuoka University, Shizuoka, Japan

Covering functors, orbit categories and derived equivalences

Let G be a group of automorphisms of a category C. We give a definition for a functor $F \colon C \to C'$ to be a G-covering and three constructions of the orbit category C/G, which generalizes the notion of a Galois covering of locally finite-dimensional categories with group G whose action on C is free and locally bonded. Here C/G is defined for any category C and does not require that the action of G is free or locally bounded. We show that a G-covering is essentially given by the canonical functor $C \to C/G$. By using this we improve a covering technique for derived equivalence.

16 Diana Avella-Alaminos

Universidad Nacional Autónoma de México, México City, México

On the derived classification of gentle algebras

(joint with: *Christof Geiss*)

The family of gentle algebras is closed under derived equivalence (by a result of Schröer and Zimmermann). The classification of this family according to derived equivalence is an interesting problem. This is well understood in the case of gentle algebras whose associated quiver has one cycle.
We study the case of two cycles. In this case we introduce certain numerical invariants of

the quiver with relations which turn out to be stable under derived equivalence.
We can show that except in a degenerate case our invariants distinguish different derived equivalence classes.

17 Michael Barot

Universidad Nacional Autónoma de México, México City, México

Generalized reflection functors, APR-tiltings and diagram mutations

In this talk we show how to generalize the reflections of Bernstein-Gelfand-Ponomarev to bounded quivers in a very combinatorial way. As we shall see, the process corresponds to APR-tilting. Furthermore, in case that the quiver does not contain an oriented cycle, we can associate to the bounded quiver a diagram (taking relations as arrows in the opposite direction) and have then that the corresponding diagrams change according to the mutation rule defined by Fomin-Zelevinsky.

18 Karin Baur

University of Leicester, Leicester, United Kingdom

Δ-filtered modules for a self-injective Nakayama algebra

(joint with: *Karin Erdmann and Alison Parker*)

We describe the class of Δ-filtered modules without self-extension for the Auslander algebra of a certain self-injective Nakayama algebra.

The motivation for this is to model the P-orbits in the nilradical of a parabolic subalgebra $\mathfrak{p} = \mathrm{Lie}P$ where P is a parabolic subgroup of SO_{2n}.

This generalises work of Bruestle, Hille, Ringel and Roehrle (1999).

19 Raymundo Bautista

Universidad Nacional Autónoma de México, Morelia, México

Monomial algebras and minimal gradable covers

A gradable quiver is a locally finite quiver Q such that in any non oriented cycle in Q, the number of arrows in one direction is the same as the number of arrows in the oposite direction.

If Q is any finite quiver, a minimal gradable covering is a covering map of quivers $q : \hat{Q} \to Q$, with $\hat{Q}$ gradable,such that if $q_0 : Q_0 \to Q$ is a covering map with Q_0 gradable, then there is a quiver map $r : Q_0 \to \hat{Q}$ such that $qr = q_0$.

Now consider an algebraically closed field k and an algebra kQ/I where I is generated by paths of length greater than one. Take $q : \hat{Q} \to Q$ a minimal gradable covering, then there is a covering morphism $kq : k\hat{Q}/\hat{I} \to kQ/I$ with $\hat{I}$ generated by paths of length greater than one. We describe $D^b(kQ/I)$ in terms of $D^b(k\hat{Q}/\hat{I})$. In particular we consider the case in which kQ/I is a gentle algebra.

20 Vladimir Bavula

University of Sheffield, Sheffield, United Kingdom

Generators and defining relations for the ring of differential operators on a smooth affine algebraic variety

For the ring of differential operators on a smooth affine algebraic variety X over a field of characteristic zero a *finite* set of algebra generators and a *finite* set of *defining* relations are found *explicitly*. As a consequence, a finite set of generators and a finite set of defining relations are given for the module $\mathrm{Der}_K(\mathcal{O}(X))$ of derivations on the algebra $\mathcal{O}(X)$ of regular functions on the variety X. For the variety X which is not necessarily smooth, a set of *natural* derivations $\mathrm{der}_K(\mathcal{O}(X))$ of the algebra $\mathcal{O}(X)$ and a ring $\mathfrak{D}(\mathcal{O}(X))$ of *natural* differential operators on $\mathcal{O}(X)$ are introduced. The algebra $\mathfrak{D}(\mathcal{O}(X))$ is a Noetherian algebra of Gelfand-Kirillov dimension $2\mathrm{dim}(X)$. When X is *smooth* then $\mathrm{der}_K(\mathcal{O}(X)) = \mathrm{Der}_K(\mathcal{O}(X))$ and $\mathfrak{D}(\mathcal{O}(X)) = \mathcal{D}(\mathcal{O}(X))$. A criterion of smoothness of X is given when X is irreducible (X is smooth iff $\mathfrak{D}(\mathcal{O}(X))$ is a simple algebra iff $\mathcal{O}(X)$ is a simple $\mathfrak{D}(\mathcal{O}(X))$-module). The same results are true for regular algebras of *essentially finite type*. For a *singular* irreducible affine algebraic variety X, in general, the algebra of differential operators $\mathcal{D}(\mathcal{O}(X))$ needs not be finitely generated nor (left or right) Noetherian, it is proved that each term $\mathcal{D}(\mathcal{O}(X))_i$ of the *order* filtration $\mathcal{D}(\mathcal{O}(X)) = \cup_{i\geq 0}\mathcal{D}(\mathcal{O}(X))_i$ is a *finitely generated left* $\mathcal{O}(X)$-module.

21 Petter Andreas Bergh

University of Oxford, Oxford, United Kingdom

(Co)homology of quantum complete intersections

(joint with: *Karin Erdmann*)

We construct a minimal projective bimodule resolution for a finite dimensional quantum complete intersection. Then we use this resolution to compute both the Hochschild cohomology and homology for such an algebra. In particular, we show that the cohomology vanishes in high degrees, while the homology is always nonzero.

22 Grzegorz Bobiński

Nicolaus Copernicus University, Toruń, Poland

On the zero set of semi-invariants for canonical algebras

Inspired by recent results of Riedtmann i Zwara concerning the zero sets of semi-invariants for quivers, we investigate the zero-sets of semi-invariants for dimension vectors of regular modules over canonical algebras. In particular, we prove that if the considered algebra is tame, then for big enough dimension vectors of regular modules the codimension of the zero-set of semi-invariants equals the number of generators of the algebra of semi-invariants. This implies that the coordinate ring of the module variety is free as a module over the algebra of semi-invariants. A similar result is obtained for a wider class of algebras, if we restrict our attention to homogeneous modules. The methods used in the proof contain a description of semi-invariants due to Skowroński/Weyman and interpretation of semi-invariants in terms of the representations of quivers.

23 Lesya Bodnarchuk

Technische Universität Kaiserslautern, Kaiserslautern, Germany

Representations of bocses and coherent sheaves on degenerations of elliptic curves

(joint with: *Yuriy Drozd*)

In my talk I shall discuss the classification problem of simple coherent sheaves on certain degenerations of elliptic curves.

Indecomposable vector bundles on smooth elliptic curves were classified in 1957 by Atiyah. In works of Burban, Drozd and Greuel it was shown that the categories of vector bundles and coherent sheaves on cycles of projective lines are tame.

It turns out, that all other degenerations of elliptic curves are vector-bundle-wild. Nevertheless, we discover that the category of coherent sheaves of any reduced plane cubic curve, including cuspidal and tacnode cubic curves and three concurrent lines, is brick-tame. The main technical tool of our approach is the representation theory of bocses. In my talk I am going to illustrate its computational potential for investigating of tame behavior in wild categories. In particular, this technique allows to prove that a simple vector bundle on a reduced cubic curve is determined by its rank, multidegree and determinant, generalizing Atiyah's classification. Our approach leads to an interesting class of bocses, which can be wild but are brick-tame.

24 Thomas Brüstle

Bishop's and Université de Sherbrooke, Sherbrooke, Canada

Gentle algebras given by surface triangulations

To every triangulation Γ of an oriented surface we associate an algebra $A(\Gamma)$. In case the triangulation does not admit self-folded triangles (or tagged arcs in the language of Fomin, Shapiro and Thurston), we show that $A(\Gamma)$ is a gentle algebra.

We give a precise description which of the algebras $A(\Gamma)$ are cluster-tilted. We also study the effect of flips in the triangulation on the corresponding gentle algebras and their representations.

This talk is based on joint work with I. Assem, G. Charbonneau and P-G. Plamondon.

25 Aslak Bakke Buan

Norwegian University of Science and Technology, Trondheim, Norway

Laurent polynomials in acyclic cluster algebras

(joint with: *Robert J. Marsh and Idun Reiten*)

Fix a quiver Q with no oriented cycles and a field k. Then there is a corresponding acyclic cluster algebra $A = A_Q$, as defined by Fomin and Zelevinsky, and a corresponding cluster category $C = C_Q$. The cluster category is a certain quotient of the derived category of the path algebra kQ and the indecomposable objects in C correspond either to indecomposable modules over kQ or to $P[1]$, where P is a projective kQ-module and [1] is the shift in the derived category.

It is known by work of Caldero and Keller [CK], see also the appendix of [BMRT], that the indecomposable exceptional objects in C correspond to the cluster variables in A, such that the correspondence maps tilting objects to clusters. Especially, as shown in [BMRT], there is a surjective map α from cluster variables to exceptional objects, with the property that a (non-polynomial) cluster variable f/m in reduced form is mapped to an exceptional object M in C. Here the composition factors of M (as a kQ-module) are determined by the monomial m, expressed in form of x_i, which are the cluster variables such that $\alpha(x_i) = P_i[1]$ for P_i indecomposable projective.

We study the more general situation where we fix an arbitrary tilting object $T = T_1 \amalg \cdots \amalg T_n$. We give sufficient and necessary conditions on T such that the result from [BMRT] generalizes, namely that if y_i is mapped to $T_i[1]$, then the denominators of the cluster varibles ($\neq y_i$) can be expressed in terms of the y_i in a way determined by the composition factors of $\mathrm{End}_C(T)$-modules coming from exceptional objects in C.

The conditions we put on T are not always satisfied. They hold for T preprojective and T preinjective. We show that for tame type, they hold if and only if T has no regular summand M in a tube of rank r, such that the quasi-length is $r - 1$.

References

[BMRT] Buan A., Marsh R., Reiten I., Todorov G. *Clusters and seeds for acyclic cluster algebras* with an appendix by Buan A., Caldero P., Keller B., Marsh R., Reiten I., Todorov G., to appear in Proc. Amer. Math. Soc.

[CK] Caldero P., Keller B. *From triangulated categories to cluster algebras II*, Annales Scientifiques de l'Ecole Normale Suprieure, 4eme serie, 39, (2006), 983-1009.

26 Igor Burban

Universität Mainz, Mainz, Germany

Cohen-Macaulay modules on non-isolated surface singularities

(joint with: *Yuriy Drozd*)

Let (R, m) be a complete local k-algebra of Krull dimension two. In works of Artin-Verdier, Auslander and Esnault it was proven that the category of maximal Cohen-Macaulay modules $\mathsf{MCM}(R)$ over R has finite representation type if and only if R is a quotient singularity. Later Dieterich, Kahn and Drozd-Greuel have shown that $\mathsf{MCM}(R)$ is tame for R minimally elliptic.

It was asked by Schreyer in 1987, whether all two-dimensional rings of discrete Cohen-Macaulay type are finite quotients of the singularities A_∞ and D_∞. It was also unknown, whether a non-isolated surface singularity can be Cohen-Macaulay tame.

In my talk I am going to show that the answer on Schreyer's question is negative. Moreover, a large class of non-isolated singularities called "degenerate cusps" (including the hypersurface singularities $k[[x, y.z]]/(xyz)$ and $k[[x, y]]/(x^2y^2)$) turns out to be Cohen-Macaulay tame.

To prove this we introduce a new class of tame matrix problems called "representations of decorated bunches of chains" generalizing the classical "bunches of (semi-) chains".

27 Juan Carlos Bustamante

Universidad San Francisco de Quito, Quito, Ecuador

Hochschild cohomology of pullbacks

(joint with: *Julie Dionne and David Smith*)

Given a finite dimensional algebra R over a field, its Hochschild cohomology groups (with coefficients in itself), which we denote by $\mathrm{HH}^i(R)$ are the groups $\mathrm{Ext}^i_{R-R}(R, R)$. In general, these groups are difficult to compute. Direct formulae exist in some particular cases, and some long exact sequences of cohomology groups have been obtained in case A admits an homological ideal, or is a triangular matrix algebra. In case A is an incidence algebra, these sequences are the Mayer-Vietoris sequences associated to some simplicial complex. However, it is known that Hochschild cohomology does not satisfy the excision axiom, so Mayer-Vietoris sequences cannot be obtained in general contexts.

The main purpose of this work is to study the Hochschild cohomology groups of a particular family of pullbacks of algebras, which we call *oriented pullbacks*. It turns out that for this class of algebras there is a Mayer-Vietoris sequence of Hochschild cohomology groups which differs from the above-mentioned sequences, and preserves the ring as well as the Gerstenhaber algebra structure on the sum $\mathrm{HH}^*(R) = \coprod_{i\geq 0} \mathrm{HH}^i(R)$. We give several examples.

28 Juan Angel Cappa

Universidad Nacional del Sur, Bahía Blanca, Argentina

The left part and the left supported algebras

(joint with: *Ibrahim Assem, María Inés Platzeck and Sonia Trepode*)

We give several characterisations of the (left) supported algebras. These characterisations are in terms of properties of the left and the right parts of the module category, or the classes L_0 and R_0 which consist respectively of the predecessors of the projective modules, and the successors of the injective modules. In connection whith this, we study the Auslander-Reiten components of an algebra A (and of its left support) which intersect the left part L_A and also the class E of the indecomposable Ext-injectives in the additive subcategory generated by L_A.

29 Jon F. Carlson

University of Georgia, Athens, USA

Modules of constant Jordan type

(joint with: *Eric Friedlander, Julia Pevtsova and Andrei Suslin*)

We introduce the class of modules of constant Jordan type for a finite group scheme G over a field k of characteristic $p > 0$. This class is closed under taking direct sums, tensor products, duals, Heller shifts and direct summands, and includes endotrivial modules. Along the way we develop some new results on nonmaximal support varieties. The class of modules of constant Jordan type together with locally split sequences forms an exact category in the sense of Quillen. We use this structure to explore realization questions.

30 Giovanni Cerulli Irelli

Università di Padova, Padova, Italy

Positivity and canonical basis in rank 3 cluster algebras of affine type

It will be explicit shown the canonical basis for some kind of rank 3 cluster algebras of affine type. It will turn out this canonical basis will be the set of the all indecomposable positive elements, with respect to a well defined and natural notion of "positivity". Moreover the canonical basis is in bijection with the root lattice of the corresponding type. In some cases it will be given an easy way to extend the canonical basis from the coefficient-free setting to the coefficient one.

31 Claudia Chaio

Universidad Nacional de Mar del Plata, Mar del Plata, Argentina

On the behavior of irreducible morphisms

In the first part of this talk we consider standard Auslander-Reiten components with trivial valuation. We study the composite of n-irreducible morphisms in these components. We give necessary and sufficient conditions for the existence of n irreducible morphisms with non zero composite in the (n+1)th-power of the radical. We show that the existence of such morphisms is in connection with the existence of cycles or bypasses in the Auslander-Reiten component.

In the second part of the talk we consider the composite of n-irreducible morphisms in regular components of type ZA_∞ or tubes. We give a description of the irreducible morphisms having this property.

32 Bo Chen

Max-Planck Institute for Mathematics, Bonn, Germany

The Gabriel-Roiter measure for hereditary algebras

For any representation-infinite hereditary algebra over an algebraically closed field, it is known that there are two kinds of partitions of the module category, namely, the take-off part, the central part and the landing part obtained by the Gabriel-Roiter measure; the preprojective, regular and preinjective by Aulander-Reiten theory. In this note, we want to compare these two kinds of partitions for tame hereditary algebras and look at how the modules are rearranged by Gabriel-Roiter measure. We also show that almost all but finitely many AR sequences terminating at Gabriel-Roiter factors are with indecomposable middle terms.

33 Claude Cibils

Université Montpellier 2, Montpellier, France

The intrinsic fundamental group of a linear category

(joint with: *Maria Julia Redondo and Andrea Solotar*)

We provide an intrinsic definition of the fundamental group of a linear category over a ring as the automorphism group of the fibre functor on Galois coverings. We prove that this group is isomorphic to the inverse limit of the Galois groups associated to Galois coverings. Moreover, the graduation deduced from a Galois covering enables us to describe the canonical monomorphism from its automorphism group to the first Hochschild-Mitchell cohomology vector space.

http://hal.archives-ouvertes.fr/hal-00155286

http://fr.arxiv.org/abs/0706.2491

34 David Craven

University of Oxford, Oxford, United Kingdom

Algebraic modules for finite groups

An algebraic module is a module that satisfies a polynomial with integer coefficients, with addition and multiplication given by direct sum and tensor product. This was defined by Alperin in the 1970s, and in 1980 Feit proved that all simple modules for p-soluble groups are algebraic.

Here we will examine the interplay between algebraic modules and the Heller operator, and more generally with the Auslander–Reiten quiver of non-projective indecomposable modules. We will briefly consider extensions to Hopf algebras.

35 Juan Cuadra

Universidad de Almeria, Almeria, Spain

Flatness in the category of comodules

For a coalgebra C over a field K the category C-$Comod$ of left C-comodules is a locally finitely presented Grothendieck category. According to Stenström, an object F in such a category is flat if every epimorphism $M \to F$ is pure. Using this notion, a left C-comodule is called flat if it is a flat object in C-$Comod$. Since in C-$Comod$ the classes of finitely presented objects and finite dimensional objects coincide, $F \in C$-$Comod$ is flat if for every epimorphism $f : M \to F$ in C-$Comod$ and every $N \in C$-$Comod$ of finite dimension the map $Hom_C(N, f) : Hom_C(N, M) \to Hom_C(N, F)$ is surjective.

In this talk we will study this notion of flat comodule. We will show several characterizations of these comodules. We will discuss the problem of the existence of enough flat comodules and the problem of the existence of flat covers in C-$Comod$. The so special features of the category of comodules (finiteness conditions, duality, etc) endow the class of flat comodules with certain distinguished characteristics, not present in other kind of categories. For instance, we will show that any flat left C-comodule may be written as a union of flat subcomodules of at most countable dimension, whenever C-$Comod$ has enough projective objects. Finally, we will deal with the following question: Assume that C-$Comod$ has enough flat comodules, has it enough projective comodules? We will give a partial answer, providing so a new characterization of one side semiperfect coalgebras. In this partial answer an interesting new class of comodules comes to the surface. The results to be presented in this talk are part of a joint work with Daniel Simson from the University of Toruń.

36 Gabriella D ' Este

Universita' di Milano, Milano, Italy

Selforthogonal modules without obvious direct summands

We describe a sufficient condition which explains the aboundance of many rather small, and not necessarily faithful, selforthogonal modules M with the following properties:

(1) The projective (resp. injective) dimension of M is finite and bigger than one.

(2) The intersection of the kernels of all covariant (resp. contravariant) Hom and Ext functors associated to M is equal to zero.

As we shall see, both "discrete" properties (that is properties of indecomposable projective-injective modules), and "continuous" properties (that is properties of more or less "visible" classes of modules) play an important role. Moreover, a kind of "cancellation" strategy of the obvious direct summands of both tilting and cotilting modules leads to rather small choices of modules M , satisfying (1) and (2), and defined over finite dimensional algebras of both finite and infinite Representation Type. We also show that left and / or right "cancellations" , as well as "additions" and "deformations" of bounded complexes (of projective modules), which are projective resolutions of partial tilting modules, turn out to be useful building blocks in the construction of bounded complexes (of projective modules) with completely different properties.

37 Julie Dionne

Université de Sherbrooke, Sherbrooke, Canada

Laura string algebras

We prove that a string algebra is laura if and only if its bound quiver does not contain a certain type of walks which we call intertwined double zero (IDOZ). While doing so, we verify also for string algebras a conjecture due to Skowronski, saying that an algebra is laura if and only if it has only finitely many isomorphism classes of indecomposable modules with both projective and injective dimension at most two. As a corollary, we also characterise the laura special biserial algebras.

38 Yuriy Drozd

National Academy of Sciences, Kiev, Ukraine

Wild algebras are controlled wild

Recall that, by Ringel, a finite dimensional K-algebra A is called *controlled wild* if for every finitely generated K-algebra Λ there is an exact faithful functor $F : \Lambda\text{-mod} \to A\text{-mod}$ such that

- If $FM \simeq FN$ then $M \simeq N$.
- If M is indecomposable, so is FM.
- There is an A-module C such that for every Λ-modules M, N

  $$\mathrm{Hom}_A(FM, FN) = F\mathrm{Hom}_\Lambda(M, N) \oplus \mathrm{Hom}_A^C(FM, FN),$$

  where $\mathrm{Hom}_A^C(X, Y)$ denotes the set of homomorphisms $X \to Y$ that factors through direct sums of copies of C.

Theorem. *Every wild algebra over an algebraically closed field is controlled wild.*

The proof uses the reduction to matrix problems (representations of boxes) and the reduction algorithm for boxes.

39 Olena Drozd-Koroleva

Kyiv Taras Shevchenko University, Kiev, Ukraine

Decorated generalized bunches of chains

We consider a class of matrix problems, which contains both generalized bunches of chains [3] and decorated bunches of chains that appear in the theory of Cohen–Macaulay modules over non-isolated singularities [2].

Definition Let $\{\mathcal{E}, \mathcal{F}, <, \sim, -, E_c, F_c\}$ be a generalized bunch of chains (GBC) over a field K in the sense of [3]. A *decoration* of this GBC consists of:

1. A discrete valuation on K and its prolongations to every skewfiled E_c, F_c; we denote by K^*, E_c^*, F_c^* the corresponding discrete valuation rings.
2. A subset D of pairs (x, y) such that $x \leq y$; we call these pairs *decorated*. Especially, we call x *decorated* if so is the pair (x, x).

The following condition must hold:

If $x \leq y \leq z$ and the pair (x, z) is decorated, then both pairs (x, y) and (y, z) are also decorated. Especially, if x occur in a decorated pair it is decorated itself.

We define *representations of a decorated GBC* by analogy with [3] and [2]. The main distinction is that for decorated pairs (x, y) the corresponding elementary transformations are only possible with coefficients from E_c^* or F_c^*. Then, following [1], we elaborate an algorithm of reduction and give a combinatorial description of indecomposable representations in terms of *strings* and *bands*.

References

[1] V. M. Bondarenko. Representations of bundles of semichained sets and their applications. St. Petersburg Math. J. 3 (1992), 973–996.

[2] I. Burban and Y. Drozd. Cohen–Macaulay modules over non-isolated surface singularities. Preprint, 2007.

[3] O. Drozd. Representations of generalized bunches of chains. Algebraic Structures and Their Applications. Kyiv, 2002, c. 224–237.

40 Alex Dugas

University of California, Santa Barbara, USA

Stable equivalence of graded algebras

(joint with: *Roberto Martinez Villa*)

Stable categories of graded modules have been connected to derived categories through Happel's work on repetitive algebras on one hand, and via a version of Koszul duality on the other. We thus extend the notion of stable equivalence to a large class of (possibly non-artinian) graded algebras. For such an algebra Λ, we focus on the Krull-Schmidt category

gr_Λ of finitely generated $\mathbb{Z}$-graded Λ-modules with degree 0 maps, and the stable category $\underline{\mathrm{gr}}_\Lambda$ obtained by factoring out those maps that factor through a graded projective module. We say that Λ and Γ are *graded stably equivalent* if there is an equivalence $\alpha : \underline{\mathrm{gr}}_\Lambda \xrightarrow{\approx} \underline{\mathrm{gr}}_\Gamma$ that commutes with the grading shift. Adapting arguments of Auslander and Reiten involving functor categories, we show that a graded stable equivalence commutes with the syzygy operator (where defined) and preserves finitely presented modules. It follows that if Λ is right noetherian (resp. right graded coherent), then so is any graded stably equivalent algebra. Furthermore, we use almost split sequences to show that a graded stable equivalence preserves finite length modules. Of particular interest in the non-artinian case, we prove that if Λ is basic with soc $\Lambda = 0$, then any graded stably equivalent basic algebra must be isomorphic to Λ as an ungraded algebra.

41 Philipp Fahr

Universität Bielefeld, Bielefeld, Germany

Coefficient quivers of regular representations of the 3-Kronecker quiver

The wild quiver having two vertices and three arrows in the same direction is known as the 3-Kronecker quiver. A useful tool to visualise a representation of a quiver is its *coefficient quiver*. I will use it to characterise some 2-parameter families of indecomposable representations of the 3-Kronecker quiver. There is a strong link between these coefficient quivers and the universal cover of the 3-Kronecker quiver giving an interesting partition of the Fibonacci numbers. The aim is to determine possible Gabriel-Roiter measures of regular representations.

42 Rolf Farnsteiner

Universität Bielefeld, Bielefeld, Germany

Support spaces, indecomposable modules and Jordan types

The study of indecomposable modules of finite group schemes of domestic representation type leads to examples such as the product $\mathrm{SL}(2)_1T_r$ of the first Frobenius kernel of SL(2) and the r-th Frobenius kernel T_r of its standard maximal torus of diagonal matrices. Understanding this module category for all $r \geq 1$ involves the investigation of the category of $\mathrm{SL}(2)_1T$-modules, which can roughly be thought of as a "covering" of $\mathrm{modSL}(2)_1T_r$.

The foregoing observations provide one motivation for the study of Jantzen's category $\mathrm{mod}G_rT$ of G_rT-modules, defined by the r-th Frobenius kernel G_r of a smooth reductive group G and a maximal torus $T \subseteq G$. This category retains nice features of $\mathrm{mod}G_r$, such as being a Frobenius category, while also being a highest weight category in the sense of Cline-Parshall-Scott. The standard objects in $\mathrm{mod}G_rT$ are the baby Verma modules $\widehat{Z}_r(\lambda) = \mathrm{Dist}(G_r) \otimes_{\mathrm{Dist}(B_r)} k_\lambda$, defined via a Borel subgroup $T \subseteq B \subseteq G$.

Work by Gordon-Green on $\mathbb{Z}^n$-graded artin algebras entails that $\mathrm{mod}G_rT$ possesses almost split sequences that are preserved by the forgetful functor $\mathfrak{F} : \mathrm{mod}G_rT \longrightarrow \mathrm{mod}G_r$. We show that representation-theoretic support spaces lead to analogs of Webb's theorem for modG_rT and modG_r, provide information on the periods of periodic modules as well as on AR-components containing modules with good filtrations.

In recent work, Friedlander-Pevtsova-Suslin have refined the notion of a support space by studying the pull-backs $\alpha^*(M) \in \mathrm{mod}k[T]/(T^p)$ of $k\mathcal{G}$-modules, defined by certain flat homomorphisms $\alpha : k[T]/(T^p) \longrightarrow k\mathcal{G}$. Here $k\mathcal{G}$ is the "group algebra" of the finite group scheme $\mathcal{G}$ over the ground field k and $p := \mathrm{Char}(k) > 0$. Building on the above, we show that Jordan types give rise to new invariants of AR-components. By way of illustration, we consider $\mathrm{SL}(2)_1T_r$-modules of constant Jordan type.

43 Elsa Fernández

Universidad Nacional de la Patagonia San Juan Bosco, Puerto Madryn, Argentina

On m-clusters and iterated tilted algebras

Let $H = kQ$ be an hereditary finite dimensional algebra over an algebraically closed field k. Consider $C_m(H)$ the m-cluster category defined to be the orbit category of D^b(mod H) under the action of the funtor $\tau^-1[m]$. Let T be an m-tilting object. We say that the algebra $C =$ End $C_m(T)$ is a m- cluster tilted algebra of type Q. In this talk we study the connections between the m-cluster tilted algebras of type Q and the iterated tilted algebras of type Q.

44 Takahiko Furuya

Tokyo University of Science, Tokyo, Japan

Hochschild cohomology of an algebra associated with a cyclic quiver

Let K be a commutative ring, s a positive integer, and $\mathit{\Gamma}$ the cyclic quiver with s vertices $e_1, ..., e_s$ and s arrows $a_1, ..., a_s$. Denote the path algebra of $\mathit{\Gamma}$ over K by $K\mathit{\Gamma}$ and the sum of all arrows in $K\mathit{\Gamma}$ by X: $X = a_1 + \cdots + a_s$. Here note that $Xe_i = a_i = e_{i+1}Xe_i$ holds for each $1 \leq i \leq s$, where we regard the subscripts i of e_i modulo s.

If K is an algebraically closed field, then it is known that any self-injective Nakayama algebra which is basic, indecomposable, and non-isomorphic to K is of the form $B_s^k := K\mathit{\Gamma}/(X^k)$ where $k \geq 2$ (see [ASS]). In [EH], Erdmann and Holm give a periodic projective bimodule resolution of B_s^k, and compute the Hochschild cohomology ring $\mathrm{HH}^*(B_s^k)$. Also, similar results have been obtained by Bardzell, Locateli and Marcos (see [BLM]).

Let n be a positive integer and $f(x)$ a monic polynomial over K of degree n. We denote the algebra $K\mathit{\Gamma}/(f(X^s))$ by A. The aim of this talk is to describe the Hochschild cohomology ring of A. In particular, if K is a field and $f(x) = x^m$ for a positive integer m in the case $ms \geq 2$, then A is the self-injective Nakayama algebra B_s^{ms}. On the other hand, if $s = 1$, then A just coincides with the algebra $K[x]/(f(x))$, and it is proved by Holm ([H]) that A has a projective bimodule resolution of period 2, and the Hochschild cohomology ring $\mathrm{HH}^*(A)$ is described.

In this talk, we show that A has a projective bimodule resolution of period 2 in the similar way in [EH] and [H] (also see [F] and [KSS]). Moreover, by means of this resolution, we determine the structure of the Hochschild cohomology group $\mathrm{HH}^t(A)$ for $t \geq 0$ as a module over the center of A. Finally, we give a presentation of the Hochschild cohomology ring $\mathrm{HH}^*(A)$ by the generators and the relations in the case $s \geq 2$ and K is a field, where the product in $\mathrm{HH}^*(A)$ is induced by the Yoneda product.

References

[ASS] I. Assem, D. Simson, A. Skowroński, *Elements of the representation theory of associative algebras*, Vol. 1. Techniques of representation theory, London Mathematical Society Student Texts 65, Cambridge University Press, Cambridge, 2006.

[BLM] M. Bardzell, A. Locateli and E. Marcos, *On the Hochschild cohomology of truncated cycle algebras,* Communications in Algebra 28 (2000), 1615–1639.

[EH] K. Erdmann and T. Holm, *Twisted bimodules and Hochschild cohomology for self-injective algebras of class A_n,* Forum Math. 11 (1999), 177–201.

[F] T. Furuya, *On an algebra associated with a circular quiver and its periodic projective bimodule resolution*, Tsukuba J. Math. 29 (2005), 247–258.

[FS] T. Furuya and K. Sanada, *Hochschild cohomology of an algebra associated with a circular quiver*, Communications in Algebra 34 (2006), 2019–2037.

[H] T. Holm, *Hochschild cohomology rings of algebras $k[X]/(f)$*, Contributions to Algebra and Geometry 41 (2000), 291–301.

[KSS] S. König, K. Sanada and N. Snashall, *On Hochschild cohomology of orders,* Arch. Math. 81 (2003), 627–635.

45 Christof Geiss

Universidad Nacional Autónoma de México, México City, México

A cluster character for preprojective algebras

(joint with: *Bernard Leclerc and Jan Schröer*)

Let Q be a quiver without oriented cycles and Λ the corresponding preprojective algebra over the complex numbers. Moreover, $\mathfrak{g} = \mathfrak{n}^- \oplus \mathfrak{h} \oplus \mathfrak{n}$ denotes the symmetric Kac-Moody algebra associated to Q.

In $\Lambda\text{-}\mathrm{mod}_0$, the category of nilpotent Λ-modules, we have a functorial isomorphism $\mathrm{Ext}^1_\Lambda(M,N) \cong D\,\mathrm{Ext}^1_\Lambda(N,M)$. This is crucial for our construction of a cluster character $\varphi\colon \Lambda\text{-}\mathrm{mod}_0 \to U(\mathfrak{n})^*$ in the sense of Y. Palu. Here, $U(\mathfrak{n})^*$ is the graded dual of the universal envelopping algebra of $\mathfrak{n}$.

φ_M is a kind of generating function for the Euler characteristics of composition series of M of all possible (fixed) types. Our construction is compatible with various nice Frobenius subcategories of $\Lambda\text{-}\mathrm{mod}_0$. Since our construction is based on Lusztig's semi-canonical basis it follows for example that cluster monomials are linearly independent.

46 Iain Gordon

University of Edinburgh, Edinburgh, United Kingdom

Quiver varieties and Cherednik algebras

(joint with: *Maurizio Martino*)

The representation theory of Cherednik algebras is quite a new field with connections to combinatorics of symmetric functions (Macdonald polynomials), algebraic symplectic geometry (resolutions and deformations of singularities), integrable systems (generalisations of Calogero-Moser systems), noetherian ring theory (differential operators), Lie theory (category O) and Hecke algebras (finite, affine and double affine!).

It turns out that Cherednik algebras seem to be a bridge between certain families of quiver varieties and finite dimensional Hecke algebras. I will explain this and how a little of the geometry of quiver varieties might be used to help to understand the representation theory of the Hecke algebras.

47 Jan Grabowski

University of Oxford, Oxford, United Kingdom

On the inductive construction of quantized enveloping algebras

I will describe an inductive scheme for quantized envelop ing algebras, arising from certain inclusions of the associated root data. These inclusions determine an algebra-subalgebra pair with the subalgebra also a quantized enveloping algebra: we want to understand the structure of the "difference" between the algebra and the subalgebra. Our point of view treats the background field and quantization parameter q as fixed and the parameter space as being the graph with vertices root data and edges given by their inclusions. More simply, one can think of the addition and deletion of nodes of Dynkin diagrams; we are interested in how the quantized enveloping algebras associated to the different diagrams are related.

By means of the Radford–Majid theorem, we see that associated to each inclusion there is a graded Hopf algebra in the braided category of modules of the subalgebra—this graded braided Hopf algebra is the object that describes the difference between the two algebras. Then using a construction of Majid called double-bosonisation, we can reconstruct the larger algebra from a central extension of the subalgebra, the graded Hopf algebra and its dual, generalising the usual triangular decomposition. I will focus on describing the structure of the graded braided Hopf algebra, showing that it is of a special kind, namely a Nichols algebra. I will also illustrate the theory with explicit examples, including the natural inclusion of $U_q(\mathfrak{sl}_3)$ in $U_q(\mathfrak{sl}_4)$. Finally, I will describe some future applications.

48 Adam Hajduk

Nicolaus Copernicus University, Toruń, Poland

A comparison of various types of degenerations for algebras

(joint with: *Piotr Dowbor*)

There are at least three different concepts of geometric degeneration for finite dimensional algebras. The classical ones, expressed in terms of orbit closures of well known action of algebraic group $\mathrm{Gl}_d(k)$ on the variety $\mathrm{alg}_d(k)$, the "rigid" ones which are radical degenerations of ordered locally bounded categories, and the so called CB-degenerations, introduced by W. W. Crawley - Boevey which are based on concept of deforming relations in finite dimensional algebra. The aim of the talk is to compare this three concepts. We present the result which states that for algebras A_0 and A_1 of same dimension, if A_0 is a CB-degeneration of A_1, then A_0 is a degeneration of A_1 in the classical sense; moreover, if additionally A_0 and A_1 are basic, then A_0 is a CB-degeneration of A_1 if and only if A_0 is a rigid degeneration of A_1. We discuss also certain result which shows that a behaviour of CB-degenerations in the different dimension case do not differ so much from that in the equal dimension case.

References

1. Piotr Dowbor, Adam Hajduk, *A comparison of various types of degenerations for algebras*, Algebra Colloquium, 14 (2007), No. 3, 361-388.

2. Adam Hajduk, *CB-degenerations and the rigid degenerations of algebras*, Colloquium Math., 106 (2006), No. 2, 305-310.

49 Yang Han

Chinese Academy of Sciences, Beijing, China

Finite-dimensional superalgebras and their representation theory

(joint with: *Deke Zhao*)

Superalgebras, either associative such as Grassmann algebras and Clifford algebras or nonassociative such as Lie superalgebras, play a quite important role not only in many mathematical branches but also in physics. Lie superalgebras have been studied extensively in recent thirty years, however the researches on associative superalgebras besides general graded algebras theory and some examples are very lack. Here we shall study in detail finite-dimensional superalgebras over algebraically closed fields and their representation theory. Our original motivation is to realize Lie superalgebras and their quantized universal enveloping superalgebras by hereditary superalgebras as what has been done for hereditary algebras via Hall algebras. This topics is also interesting in its own right. Indeed, Gordon and Green have been studied torsionfree group-graded algebras, it is natural to consider finite group-graded algebras, in particular superalgebras and their representation theory.

Firstly, we shall reduce the representation theory of a finite-dimensional superalgebra to that of a finite-dimensional basic superalgebra via graded equivalent theory. Secondly, we shall reduce the representation theory of a finite-dimensional basic superalgebra to that of a superspecies with super relations. Thirdly, we shall reduce the representation theory of a superspecies to that of its associated quiver. Fourthly, we shall obtain the classification of superspecies according to their representation type and the representation theory of representation finite superspecies and tame superspecies. Finally, we shall construct Hall superalgebras and show that they are Hopf sueralgebras.

50 Dieter Happel

Technische Universität Chemnitz, Chemnitz, Germany

Reconstruction of path algebras from their posets of tilting modules

(joint with: *Luise Unger*)

Let $\overrightarrow{\Delta}$ be a finite quiver without oriented cycles and $n \geq 2$ vertices. For a field k we denote by $k\overrightarrow{\Delta}$ the path algebra of $\overrightarrow{\Delta}$ over k.

If T is a $k\overrightarrow{\Delta}-$tilting module (i.e. $\mathrm{Ext}^1_{k\overrightarrow{\Delta}}(T,T) = 0$ and the number of non isomorphic indecomposable direct summands of T equals n) we denote by $\mathcal{G}(T)$ the full subcategory of $\mathrm{mod}\, k\overrightarrow{\Delta}$ containing those $k\overrightarrow{\Delta}-$modules which are epimorphic images of copies of T.

Let $\mathcal{T}_{k\overrightarrow{\Delta}}$ be the poset of $k\overrightarrow{\Delta}-$tilting modules. The elements of $\mathcal{T}_{k\overrightarrow{\Delta}}$ are the isomorphism classes of multiplicity free $k\overrightarrow{\Delta}-$tilting modules and for $T, T' \in \mathcal{T}_{k\overrightarrow{\Delta}}$ we set $T' \leq T$ if $\mathcal{G}(T') \subseteq \mathcal{G}(T)$.

For a quiver $\overrightarrow{\Delta}$ the decorated underlying simply-laced quiver $\overrightarrow{\Delta}_{\mathrm{dec}}$ has the same vertices as $\overrightarrow{\Delta}$. If for a pair of vertices (i, j) of $\overrightarrow{\Delta}$ there are $r \geq 1$ arrows from i to j in $\overrightarrow{\Delta}$, then there is one arrow from i to j in $\overrightarrow{\Delta}_{\mathrm{dec}}$ if $r = 1$ and one decorated arrow $i \xrightarrow{*} j$ in $\overrightarrow{\Delta}_{\mathrm{dec}}$ if $r \geq 2$.

The main result asserts that in case $\overrightarrow{\Delta}$ does not contain isolated vertices, then the poset $\mathcal{T}_{k\overrightarrow{\Delta}}$ determines $\overrightarrow{\Delta}_{\mathrm{dec}}$ up to isomorphism.

51 Anne Henke

University of Oxford, Oxford, United Kingdom

Brauer algebras and Schur-Weyl dualities

(joint with: *Robert Hartmann, Steffen König and Rowena Paget*)

Let n and r be natural numbers. In 1937, Brauer asked the following question: which algebra has to replace the group algebra of the symmetric group $\mathbb{C}\Sigma_r$ in the set-up of "Schur-Weyl duality" if one replaces the general linear group GL_n by its orthogonal (or symplectic) subgroup. As an answer he defined an algebra which is a special case of what nowadays is called the Brauer algebra $B_k(r, \delta)$.

Let k be a field of prime characteristic, $\delta \in k$ and E an n-dimensional vector space. We completely describe the tensor space $E^{\otimes r}$ viewed as a module for the Brauer algebra $B_k(r, \delta)$ with parameters $\delta = 2 = n$. It turns out that:

(1) the tensor space typically is not filtered by cell modules, and thus will not be equal to a direct sum of Young modules as defined in Hartmann and Paget. This is very different from the situation for group algebras of symmetric groups.

(2) the tensor space in this situation still affords Schur-Weyl duality.

Let Y be a direct sum of Young modules (of the Brauer algebra), let $S = End_{B(r,\delta)}(Y)$. Then Schur-Weyl duality holds between S and $B(r, \delta)$, afforded by the bimodule Y.

52 Martin Herschend

Uppsala University, Uppsala, Sweden

On the Clebsch-Gordan problem for quiver representations

Given a Krull-Schmidt category equipped with a tensor product one can pose the Clebsch-Gordan problem, i.e. the problem of finding the decomposition of any two objects into a direct sum of indecomposables. It arose from the theory of group representations, the classical case being representations of $SU(2)$.

We study the Clebsch-Gordan problem for quiver representations, where the tensor product is defined point-wise and arrow-wise. Over an algebraically closed ground field of characteristic 0 this problem has been solved for the loop $\tilde{\mathbb{A}}_0$ by Huppert and independently by A. Martsinkovsky and A. Vlassov. It has also been solved for some representations of the four factorspace quiver $\tilde{\mathbb{D}}_4$ by E. Dieterich in connection with his investigation of lattices over curve singularities. I will present the solution for Dynkin quivers of type $\mathbb{A}_n$, $\mathbb{D}_n$ and $\mathbb{E}_6$ with focus on describing the corresponding representation rings.

I will also present results describing the behaviour of the point-wise tensor product under Galois coverings. These are applied to find the solution for extended Dynkin quivers of type $\tilde{\mathbb{A}}_n$ and the double loop quiver with relations $\alpha\beta = \beta\alpha = \alpha^n = \beta^n = 0$. The latter was originally studied by I. M. Gelfand and V. A. Ponomarev in their investigation of representations of the Lorentz group.

53 Lutz Hille

Universität Bielefeld, Bielefeld, Germany

Fans and the volume of a tilting module

It is well-known that the set of all tilting modules defines a simplicial set in a natural way. Thus there is a topological space associated to any finite dimensional algebra: the geometric realization of the simplicial complex of tilting modules. If we consider tilting modules of projective dimension at most one, this simplicial complex has further natural structures: to each tilting module we can associate a cone in the Grothendieck group, each indecomposable rigid module defines a lattice point and two different cones intersect only along the boundary. Moreover, we can use these cones to define the volume of a tilting module. Informally, it is a measure for the size of a tilting module. Also the coefficients of the middle term for the mutation sequences are contained in the lattice geometry associated to these cones. Finally, if we intersect the cones with the unit ball, we obtain the geometric realization of this simplicial complex naturally embedded in the reell Grothendieck group.

We briefly describe our main results: the cones of the tilting modules form a so-called fan (informally a set of cones satisfying certain conditions, it is the main notion in toric geometry). Provided, there exists a rigid module for each dimension vector, the sum of the volumes taken over all isomorphism classes of basic tilting modules is one. This result also generalizes to tame quivers. For any algebra this sum is at most one. Such a volume formula is very useful to check, whether a given list of tilting modules is already complete.

As an application we are able to compute this fan explicitly for many examples and can associate similar notions to cluster categories.

54 Thorsten Holm

Universität Magdeburg and University of Leeds, Magdeburg/Leeds, Germany/United Kingdom

Cluster categories and selfinjective algebras

(joint with: *Peter Jørgensen*)

Cluster categories have been introduced as a categorification of the cluster algebras of Fomin and Zelevinsky, and have quickly become an independent and very active branch of representation theory. The cluster categories, and the more general u-cluster categories are defined as orbit categories of the bounded derived category of hereditary algebras, modulo the functor $\tau^{-1}\Sigma^u$ where τ is the Auslander-Reiten translation and Σ the suspension.

It has been shown by Keller that u-cluster categories are triangulated, and recently by Keller and Reiten that among triangulated categories, u-cluster categories are very natural objects: there is a Morita theorem to the effect that any triangulated category of algebraic origin which shares the formal properties of a u-cluster category of type kQ is actually triangulated equivalent to such a category.

Based on this seminal Keller-Reiten theorem we were able to establish a connection between cluster theory and selfinjective algebras by proving that certain u-cluster categories of Dynkin types $\Delta \in \{A_n, D_n, E_n\}$ are triangulated equivalent to the stable module categories of selfinjective algebras of finite representation type, of corresponding tree class Δ.

A crucial aspect of the proof is to show that the stable module categories under consideration have the 'correct' Calabi-Yau dimension. To do this, we proved, using recent results of Amiot, that the stable Calabi-Yau dimension of a selfinjective algebra of finite type is determined by the action of the Nakayama and suspension functors on objects.

(Preprints at arXiv:math.RT/0610728, math.RT/0612451, math.RT/0703488.)

55 Angela Holtmann

Universität Bielefeld, Bielefeld, Germany

Tame dimension vectors for wild quivers

It is well known that the indecomposable representations of wild quivers depend on arbitrary many parameters. So the classification of all indecomposable representations for wild quivers is impossible. Nevertheless, one may ask for those dimension vectors which show a somewhat "tame" behaviour.

A dimension vector $\mathbf{d}$ is called *tame* if there is a one parameter family of indecomposable representations for $\mathbf{d}$ and, given any (pointwise) decomposition $\mathbf{d} = \mathbf{d_1} + \mathbf{d_2}$ as a sum of two dimension vectors, all families of indecomposable representations for both $\mathbf{d_1}$ and $\mathbf{d_2}$ depend on at most one parameter.

The second condition implies that there is no decomposition $\mathbf{d} = \mathbf{d_1} + \mathbf{d_2}$ in which $\mathbf{d_1}$ or $\mathbf{d_2}$ is a so called "hypercritical" dimension vector. (The *hypercritical* dimension vectors are the minimal ones for which there is an m-parameter family of indecomposable representations with $m \geq 2$.) Although there are infinitely many quivers, the latter ones have been classified —there is a complete (finite) list— and can also be characterised in completely combinatorial terms as those dimension vectors $\mathbf{d}$ whose Tits form is negative, but non-negative on all dimension vectors $\mathbf{d_1}$ and $\mathbf{d_2}$ occurring in non-trivial decompositions $\mathbf{d} = \mathbf{d_1} + \mathbf{d_2}$.

56 Alicja Jaworska

Nicolaus Copernicus University, Toruń, Poland

On components of the Auslander-Reiten quiver of trivial extensions of 2-fundamental algebras which contain projective modules

One of the classes of algebras whose Auslander-Reiten quiver has not been known yet is a class of tame algebras which are not of polynomial growth. We shall study some algebras from this class. The minimal 2-fundamental algebras were introduced in [4]. Then Auslander-Reiten quiver of their trivial extension were consider in [3]. There were some results on starting and ending components of nonpolynomial growth algebras given.

We will continue these examinations. Let A be a minimal 2-fundamental algebra from one of the families given in [3]. Then a trivial extension $T(A)$ is tame algebra but not of polynomial growth. We are especially interested in the components of $\Gamma_{T(A)}$ which contain indecomposable projective $T(A)$-modules. We aim to provide the most important information such as the structure of these components, their number and position in relation to each other. It emerges that all indecomposable projective $T(A)$-modules lie in nonstable tubes or in components of pseudotype $\mathbb{Z}\mathbb{A}_{\infty}^{\infty}$, which are not generalized standard. All obtained properties allow us to construct a full subgraph of a components graph. It is a first step in approaching to such an awaited Auslander-Reiten quiver.

References:

1. M.Auslander, I.Reiten, *Uniserial functors*, LMN 832 (1980), p.1-47.

2. Ch.Geiss, *On components of type $\mathbb{Z}\mathbb{A}_\infty^\infty$ for string algebras*, Comm. Algebra, 26(3), (1998), p.749-758.

3. A.Jaworska, Z.Pogorzały, *On trivial extensions of 2-fundamental algebras*, Comm. Algebra, 34(2006), no.11, p. 3935-3947.

4. Z.Pogorzały, M.Sufranek *Starting and ending components of the Auslander-Reiten quivers of a class of special biserial algebras*, Colloq. Math., 99(1), p.111-144.

5. A.Skowroński, *Algebras of polynomial growth*, Banach Center Publications Vol. 26 Part I (Warszawa, 1990), p.535-568.

6. A.Skowroński, *Generalized standard Auslander-Reiten components*, J. Math Soc. Japan 46(1994), p. 517-543.

7. A.Skowroński, J.Waschbüsch, *Representation-finite biserial algebras*, J. reine angew. Math., 345 (1983), p. 172-181.

57 Peter Jørgensen

University of Newcastle, Newcastle, United Kingdom

Calabi-Yau categories and Poincaré duality spaces

Let X be a topological space. The singular cochain complex $C^*(X)$ is a differential graded algebra which has a derived category D.

The compact part D^c of D has several nice properties. The following ones will b e explained in this talk:

Theorem. The category D^c is d-Calabi-Yau if and only if X is a d-di mensional Poincaré duality space.

Theorem. Let X be a d-dimensional Poincaré duality space. Then the Auslander-Reiten quiver of D^c consists of copies of $\mathbb{Z}A_\infty$.

58 Stanisław Kasjan

Nicolaus Copernicus University, Toruń, Poland

A remark on periodicity in representation theory of finite dimensional algebras

Let Λ be an algebra over an algebraically closed field K and K has infinite transcendence degree over its prime subfield. Fix a natural number s. We prove that if every s-dimensional indecomposable nonprojective Λ-module is Ω-periodic (resp. DTr-periodic) then there exists a common bound for the Ω-periods (resp. DTr-periods) of s-dimensional indecomposable nonprojective Λ-modules. This is done by applying some standard model-theoretic arguments.

59 Shigeto Kawata

Osaka City University, Osaka, Japan

Heller lattices and Auslander-Reiten quivers for integral group rings

Let G be a finite group, $\mathcal{O}$ a complete discrete valuation ring of characteristic zero with residue class field $k = \mathcal{O}/\pi\mathcal{O}$ of characteristic $p > 0$, and B a block of the group ring $\mathcal{O}G$. For a kG-module M, the kernel Z of the projective cover P of M viewed as an $\mathcal{O}G$-module is called the *Heller $\mathcal{O}G$-lattice* of M: $0 \to Z \to P \to M$ (exact). Here we assume that $\mathcal{O}$ is sufficiently large to satisfy certain conditions.

First, we see that the Heller $\mathcal{O}G$-lattices of the indecomposable kG-modules are indecomposable. Also, we show that the almost split sequences of kG-modules are, modulo direct summands of split sequences, liftable to almost split sequences of $\mathcal{O}G$-lattices terminating in Heller $\mathcal{O}G$-lattices.

Next, we suppose moreover that a block B of $\mathcal{O}G$ is of infinite representation type. Let Θ be a connected component of the stable Auslander-Reiten quiver of B containing some Heller $\mathcal{O}G$-lattices. Then we see that the tree class of Θ is A_∞ and Heller lattices lie at the end of Θ. Also, it follows that B has infinitely many components of type $\mathbf{Z}A_\infty$ if a defect group of B is neither cyclic nor a Klein four group.

60 Bernhard Keller

Université Paris 7, Paris, France

K-theory via universal invariants

(joint with: *Gonçalo Tabuada*)

This is a report on chapter 3 of G. Tabuada's Ph. D. Thesis. The main result is the interpretation of higher algebraic K-theory as a space of morphisms in the target category of the universal additive invariant of dg categories. Essentially, this invariant is the universal functor from the category of differential graded categories with values in a triangulated category which transforms derived Morita equivalences into isomorphisms and semi-orthogonal decompositions into direct sums. The correct formulation of this property and the proof of the main theorem are based on the formalism of derivators as developped by Heller, Grothendieck, the reporter, Maltsiniotis and Cisinski.

61 Otto Kerner

Heinrich-Heine-Universität Düsseldorf, Düsseldorf, Germany

Cluster tilted algebras of rank three

Let H be a connected hereditary algebra of rank three, T a squarefree tilting H-module and $\Gamma = \mathrm{End}_{\mathcal{C}}(T)$ the endomorphism ring of T in the cluster category $\mathcal{C}_H$ of H. The algebra Γ is called *cluster tilted algebra* of rank three, or of type H.

In [1] a new characterisation of cluster tilted algebras is given: An algebra Γ is cluster tilted (of type H) if and only if there exists a tilted algebra B (of type H) such that

$\Gamma \cong B \ltimes \mathrm{Ext}^2_B(DB, B)$. If Γ is cluster tilted of rank three and not hereditary, its quiver $\mathcal{Q}$ has the following shape:

$$\begin{tikzcd} & \mathbf{3} \arrow[dr, "r"] & \\ \mathbf{1} \arrow[ur, "t"] & & \mathbf{2} \arrow[ll, "s"'] \end{tikzcd}$$

The symbol $x \xrightarrow{m} y$ means that there are m arrows from x to y. It is shown in [2], which of these cyclic quivers are the quivers of cluster tilted algebras.

If H is representation infinite, and the cluster tilted algebra Γ is not hereditary, then the Loewy-length of Γ is $3 + r$, where r is the number of regular indecomposable direct summands of T [3]. It will be demonstated, how the number r can be deduced from the shape of the quiver $\mathcal{Q}$.

References

[1] I. Assem, T. Brüstle and R. Schiffler, *Cluster-tilted algebras as trivial extensions*, to appear in Bull. London Math. Soc.

[2] A. Beineke, T. Brüstle and L. Hille, *Cluster-cyclic quivers with three vertices and the Markov equation*, Preprint, Dec. 2006.

[3] O. Kerner, *Wild cluster-tilted algebras of rank three*, to appear in JPAA.

62 Ahmed Khammash

Umm AlQura University, Makkah, Saudi Arabia

I-sequences in the category of modules

This note is a quick report on the notion of I-sequences which generalizes the notions of almost split sequences and relative projective covers for modules. It is shown that, by using this notio, many well-known results on almost split sequences and relative projective covers can be recovered in a slight generalization.We also discuss the notion of generalized Heller functor associated with ideals in the category of modules which is a generalization of the known Heller operator. We study some of the main properties of the generalized Heller operator and raise some open questions in this direction.

63 Mark Kleiner

Syracuse University, Syracuse, USA

A parametrization of preprojective representations of quivers, Gabriel's theorem, and the Weyl group

According to Gabriel's theorem, a connected quiver is of finite representation type if and only if its underlying graph is of Dynkin ADE type. In this case, the indecomposable representations are parametrized by their dimension vectors, which correspond bijectively to the positive roots of the associated root system. Since every representation of a quiver of ADE type is preprojective, one may ask how to parametrize the indecomposable preprojective representations of any connected quiver without oriented cycles by objects related to the associated Kac-Moody algebra. We give such a parametrization by the elements of a canonical subset of the Weyl group, which we construct using only the geometry of the quiver. This involves an analysis of a shortest sequence of Bernstein-Gelfand-Ponomarev (sink) reflection functors whose composition annihilates a given preprojective representation, and a study of the corresponding composition of simple reflections in the Weyl group. A consequence is that the Weyl group is infinite if and only if the powers of any Coxeter element are reduced words. Speyer modified our proof and showed that the latter holds for any irreducible Coxeter group. The talk is based on joint work with Allen Pelley and Helene Tyler.

64 Stefan Kolb

University of Edinburgh, Edinburgh, United Kingdom

The center of quantum symmetric pair coideal subalgebras

(joint with: *Gail Letzter*)

Quantum symmetric pairs, i.e. quantum group analogs of $U(\mathfrak{g}^\theta)$ for symmetric semisimple Lie algebras $(\mathfrak{g}, \theta)$, were constructed by M.Noumi, G. Letzter and others. As $\mathfrak{g}^\theta$ is reductive one expects a representation theory also in the quantum case. As a first step in this direction we determine the center of these algebras, showing that it is a polynomial ring in the right number of variables.

65 Henning Krause

Universität Paderborn, Paderborn, Germany

Localising subcategories of the stable module category of a finite group

(joint with: *Srikanth Iyengar and Dave Benson*)

We classify the localising subcategories of the stable module category for a finite group. This enables us to prove the telescope conjecture in this context, as well as give a new proof of the tensor product theorem for support varieties.

In my talk I explain the history of this classification problem as well as the strategy of our proof. The challenge is basically to reduce this classification to a problem from commutative algebra. The main tools are support varieties and local cohomology. Then we use a similar classification of localising subcategories for the derived category of a commutative noetherian ring, which Neeman obtained some 15 years ago.

66 Dirk Kussin

Universität Paderborn, Paderborn, Germany

The cluster category of a canonical algebra

(joint with: *Michael Barot and Helmut Lenzing*)

Let k be an algebraically closed field. We study the cluster category $\mathcal{C}(A)$ of a canonical algebra A, or equivalently, the cluster category of the hereditary category $\mathcal{H}$ of coherent sheaves on a weighted projective line. This point of view gives a slightly simplified approach to $\mathcal{C}(A)$ and allows an elegant treatment of, for example, the tame hereditary case, the tubular case, etc. By a theorem of B. Keller there is a triangulated structure on $\mathcal{C}(A)$ such that the canonical projection functor $\pi : \mathcal{D}^b(A) \to \mathcal{C}(A)$ is exact.

We focus on the description of the Grothendieck group of $\mathcal{C}(A)$ for an arbitrary canonical algebra, and on a more detailed study of the category $\mathcal{C}(A)$ for tubular A. In particular, in the tubular case we determine the group of (isoclasses of) exact autoequivalences of $\mathcal{C}(A)$.

67 Marta Kwiecień

Nicolaus Copernicus University, Toruń, Poland

Selfinjective algebras of quasitilted type with almost all simple modules periodic

A finite dimensional algebra A over an algebraically closed fileld K is said to be a selfinjective algebra of quasitilted type if A is the orbit algebra $\widehat{B}/G$ of the repetitive algebra $\widehat{B}$ of a quasitilted algebra B with respect to the action of an admissible infinite cyclic group G of automorphisms of $\widehat{B}$. We will describe the structure of all representation-infinite selfinjective algebras of quasitilted type having almost all simple modules periodic.

68 Daniel Labardini

Northeastern University, Boston, USA

Quivers with potentials associated to triangulated surfaces

Recently, H.Derksen, J.Weyman and A.Zelevinsky have introduced quivers with potentials and their mutations, thus providing a new representation-theoretic interpretation for quiver mutations originated in the theory of cluster algebras. In another recent development, S.Fomin, M.Shapiro and D.Thurston have introduced a class of quivers associated with triangulated oriented bordered surfaces with marked points. They showed that this class is invariant under quiver mutations, which correspond to geometrically defined flips on ideal triangulations. In this talk we discuss the possibility of explicitly lifting these flips to the level of quivers with potentials.

69 Sefi Ladkani

Hebrew University of Jerusalem, Jerusalem, Israel

Posets, sheaves, and their derived equivalences

A finite partially ordered set (poset) X can be naturally endowed with a structure of a topological space. Hence, given a fixed field k, we can speak of sheaves of k-vector spaces over X. These sheaves can be identified with certain commutative diagrams and also with modules over the incidence algebra of X.

We say that two posets are *derived equivalent* if their bounded derived categories of sheaves are equivalent. This gives an equivalence relation between finite posets, which is coarser than isomorphism, but still fine enough to be interesting. However, there is no known algorithm that determines for two posets X and Y whether they are derived equivalent or not.

We will present two constructions that produce, given posets having certain combinatorial properties, new posets which are derived equivalent to them. The common theme of these constructions is the structured reversal of order relations.

The first is the "bipartite construction". We show that any poset having a bipartite structure is derived equivalent to the poset obtained by mirroring with respect to that structure. More generally, any triangular matrix algebra satisfying some finiteness conditions has a natural mate – another triangular matrix algebra derived equivalent to it.

The second construction produces, for pairs of posets sharing common underlying combinatorial structure, *universal* derived equivalences, that is, derived equivalences of sheaves over any abelian category. Our main tools are formulas, which are pieces of combinatorial data allowing us to define functors between the categories of complexes, inducing triangulated functors between the derived categories.

This last construction vastly generalizes the well-known BGP reflection functors on quivers and has applications to some partial orders of cluster tilting objects in cluster categories.

70 Sefi Ladkani

Hebrew University of Jerusalem, Jerusalem, Israel

Piecewise hereditary algebras and posets

A piecewise hereditary algebra is a finite dimensional algebra whose derived category of modules is equivalent to a derived category of an abelian category of global dimension one. The two main examples are path algebras of quivers and the canonical algebras.

We will present two results concerning the global dimension and the Euler form of piecewise hereditary algebras.

First, we give bounds on the global dimension of a finite length, piecewise hereditary category in terms of quantitative connectivity properties of its graph of indecomposables. We use this to show that the global dimension of a finite dimensional, piecewise hereditary algebra A cannot exceed 3 if A is an incidence algebra of a finite poset or more generally, a sincere algebra. This bound is tight.

Second, we show that for piecewise hereditary algebras, the periodicity of the Coxeter transformation implies the nonnegativity of the Euler form. The condition of piecewise heredity cannot be omitted, even for triangular algebras, as demonstrated by incidence algebras of finite posets.

71 Patrick Le Meur

École Normale Supérieure de Cachan, Cachan, France

Universal cover and simple connectedness of piecewise hereditary algebras

Galois coverings were introduced in the eighties by Riedtmann and by Bongartz and Gabriel in order to reduce the study of a finite dimensional algebra A to the study of an algebra which is easier to handle. This has led to the property of simply connectedness for a triangular algebra (due to Assem and Skowroński) which characterises the fact that the study of A cannot be reduced to a strictly easier one using a Galois covering. It is therefore natural to look for a simple characterisation of simple connectedness. Skowroński asked in 1993 whether a tame triangular algebra A is simply connected if and only if $HH^1(A) = 0$. This equivalence was proved for A tilted of type Q if Q is euclidean (Assem and Skowroński) or if A is tame (Assem, Marcos and de la Peña). Also, it was proved for A tame quasitilted (Assem, Coelho and Trepode). In this talk, we shall prove that if A is piecewise hereditary of type a quiver Q (without assumption on the representation type of A and Q), then A is simply connected if and only if $HH^1(A) = 0$. This will be done by proving that A admits a Galois covering with group $\pi_1(Q)$ and which is universal with respect to the Galois coverings of A. Also, we shall show the same characterisation of simple connectedness for A quasitilted of any representation type.

72 Shiping Liu

Université de Sherbrooke, Sherbrooke, Canada

Auslander-Reiten theory in a Krull-Schmidt category

Let $\mathcal{A}$ be a Hom-finite Krull-Schmidt category over a commutative artinian ring. We introduce the notion of an Auslander-Reiten sequence in $\mathcal{A}$. This provides an unified treatment of the Auslander-Reiten theory in various categories: abelian categories, exact categories, and triangulated categories. In case $\mathcal{A}$ has Auslander-Reiten sequences, the Auslander-Reiten components of $\mathcal{A}$ look like those of an artin algebra with a few exception. In particular, our results can be applied to study the Auslander-Reiten components of the derived category of an artin algebra of finite global dimension.

73 Yuming Liu

Universität Köln, Köln, Germany

Gluing of idempotents, radical embeddings and two classes of stable equivalences

(joint with: *Steffen König*)

Stable equivalences are studied between any finite dimensional algebra A with a simple projective module and a simple injective module and an algebra B obtained from A by 'gluing' the corresponding idempotents of A, extending results by Martinez-Villa. Stable equivalences modulo projectives are compared to stable equivalences modulo semisimples, and in either situation a characterisation is given for a radical embedding to induce such a stable equivalence.

74 Javier López Peña

University of Granada, Granada, Spain

On the classification of factorization structures of low dimension

(joint with: *Gabriel Navarro*)

We give an explicit description of the set of all factorization structures, or twisting maps, existing between the algebras k^2 and k^2, and classify the resulting algebras up to isomorphism. In the process we relate several different approaches formerly taken to deal with this problem, filling a gap that appeared in a recent paper by Cibils, and providing a counterexample to a result concerning the Hochschild (co)homology appeared in a paper by J.A. Guccione and J.J. Guccione. We also give a combinatorial approach to the description of all factorization structures for which one of the factors is an algebra of dimension 2.

75 Dag Madsen

Norwegian University of Science and Technology, Trondheim, Norway

Degeneration of A-infinity modules

(joint with: *Bernt Tore Jensen and Xiuping Su*)

We use A-infinity modules to study the derived category of a finite dimensional algebra over an algebraically closed field. There are varieties parameterising A-infinity modules with given homology dimensions. These varieties carry an action of an algebraic group such that orbits correspond to quasi-isomorphism classes of complexes in the derived category (with the given homology dimensions). We describe orbit closures in these varieties, generalising the Riedtmann-Zwara Theorem.

76 Eduardo Marcos

Universidade de São Paulo, São Paulo, Brasil

$2 - D$ determined algebras

(joint with: *Ed Green*)

In this talk, we describe some classes of graded algebras, which we call $2-d$ determined. These are algebras whose ideal of relations have generators in degree 2 and d. In some cases the *Ext*-algebra associated, is generated in degrees 1 and 2. This is the beginning of a joint work.

77 Andrei Marcus

"Babeş-Bolyai" University Cluj-Napoca, Cluj-Napoca, Romania

Characters and equivalence classes of central simple group graded algebras

An equivalence relation between central simple G-acted F-algebras was introduced by A. Turull in [5] in order to study Schur indices of irreducible complex characters of finite groups in the context of Clifford theory.

In [3] we consider strongly graded G-algebras $A = \bigoplus_{g\in G} A_g$, where G is a finite group and A is a finite dimensional F-algebra having no non-trivial G-graded ideals. Then G acts on $Z(A_1)$, and we assume that $Z(A_1)^G = F$. We call such an F-algebra a central simple crossed product of A_1 and G.

We say that two G-graded central simple crossed products are equivalent if there is a G-graded Morita equivalence (see [2]) between them. We show that this relation comes down to Turull's equivalence relation in the particular case of central simple G-acted F-algebras. Moreover, even the treatment of the particular case is easier when we bring crossed products into the discussion.

We study invariants of this equivalence relation, and also the structure of certain representatives of the equivalence classes.

Let χ be an absolutely irreducible character of a semisimple strongly G-graded F-algebra R and let η be the restriction of χ to R_1. Then we associate to χ an equivalence class of central simple crossed products over $F(\eta)$, and we investigate the situation when the classes of two characters are equal.

References

[1] Herman, A.: Clifford theory with Schur indices in projective representation theory. Comm. Algebra **32** (2004), 3797–3806.

[2] Marcus, A.: Equivalences induced by graded bimodules. Communications in Algebra **26** (1998), 713–731.

[3] Marcus, A.: Characters and equivalence classes of central simple crossed products. Preprint 2006, to appear in Comm. Algebra.

[4] Riese, U. and P. Schmid, Schur indices and Schur groups II. J. Algebra **182** (1996), 183–200.

[5] Turull, A.: Clifford theory with Schur indices. J. Algebra **170** (1994), 661–677.

[6] Turull, A.: Equivalence of G-algebras with complemented centroid. Comm. Algebra **22** (1994), 5037–5078.

78 Robert Marsh

University of Leeds, Leeds, United Kingdom

Catalan sets, Pascal arrays, and towers of algebras

(joint with: *Paul Martin*)

The Bratelli diagram of a tower $\Lambda_0, \Lambda_1, \ldots$ of semisimple algebras encodes the composition factors of the restriction of a simple module over Λ_n to Λ_{n-1} for each n. If Λ_0 is 1-dimensional, walks on the diagram correspond to basis elements of the modules. In well-behaved cases such walks can be modelled via an underlying graph (the Rollet graph).

Motivated by the Temperley-Lieb algebra and other examples, we define a set of axioms for a tower of algebras together with collections of modules which guarantee the existence of a Rollet graph. There are some interesting examples of towers satisfying these axioms, including the Brauer algebras, the partition algebras and contour algebras.

We also show that this extra structure underlies a variety of interesting sequences of combinatorial sets. In particular, the clusters of the Fomin-Zelevinsky cluster algebras of type A_n, $n = 1, 2, \ldots$, form such a sequence.

79 Roberto Martinez Villa

Universidad Nacional Autónoma de México, Morelia, México

Artin Schelter regular algebras/categories and AR-theory

(joint with: *Øyvind Solberg*)

We define and generalize the class of Artin-Schelter regular algebras. Some central properties of these algebras are recalled, and our concept of an AS-regular algebra is modelled on this behavior. The generalization is motivated through considering the class of Auslander algebras for finite dimensional algebras. This is further generalized to categories and applied to study the regular (preprojective/preinjective) components of a finite dimensional algebra. Finally we discuss how the shape of the component is related to properties of the naturally associated AS-regular Koszul category constructed in the lecture *Koszul theory for all finite dimensional algebras* by Øyvind Solberg.

80 Maurizio Martino

Universität Bonn, Bonn, Germany

Restricted rational Cherednik algebras and two-sided cells

(joint with: *Iain Gordon*)

Rational Cherednik algebras are an interesting class of infinite dimensional algebras which appear in many areas of mathematics such as representation theory, combinatorics and algebraic geometry. They are defined for any finite complex reflection group (and so in particular any Weyl group) and are deformations of smash product algebras arising in the invariant theory of these groups. Many properties of these algebras can be deduced from certain finite dimensional quotient algebras which are called restricted rational Cherednik algebras. We shall explain a bijection between the blocks of these algebras and two-sided cells in the case of the Weyl group of type B_n. Cells are subsets of the Weyl group defined via the corresponding Hecke algebra and play a fundamental role in the representation theory of p-adic groups of Lie type and the geometry of Schubert varieties. We shall state a conjecture which generalises and extends this result to other types.

81 Volodymyr Mazorchuk

Uppsala University, Uppsala, Sweden

Algebraic categorification

The term *categorification* was introduced in 1995 by Louis Crane based on ideas developed jointly with Igor Frenkel. Categorification is the process of finding category-theoretical analogs of set-theoretical concepts by e.g. replacing sets with categories, functions with functors, and equations between functions by isomorphisms between functors. The most famous result, which uses the idea of categorification seems to be Khovanov's categorification of the Jones polynomials (an invariant of knots and links).

Algebraically, the notion *categorification* is not uniquely defined yet. As in topology, it was used as a kind of philosophy by several authors. Nevertheless, there are some spectacular examples and interesting applications. For instance, a (very strong) version of sl_2-categorification was used by Chuang and Rouquier to prove Broué's abelian defect group conjecture for symmetric groups. Different suggestions for the algebraic definition of categorification are discussed in the literature.

The aim of this talk is to present the algebraic version of the categorification philosophy, starting from the discussion of different definitions (and even levels of definitions) of categorification. All definitions will be illustrated by examples and accompanied by a discussion of naturality, advantages and disadvantages. Examples vary from categorifications using abelian categories to categorifications using additive or triangulated categories and include certain modules over the symmetric group, braid group, quantum groups, Weyl algebra etc. A special emphasize will be made on already known and potential applications.

82 Hagen Meltzer

University of Szczecin, Szczecin, Poland

An algorithm for exceptional modules over tubular canonical algebras

(joint with: *Piotr Dowbor and Andrzej Mróz*)

This is a report on joint work with Piotr Dowbor and Andrzej Mróz. We describe an algorithm and present a computer program for exceptional modules over tubular canonical algebras. The input for the algorithm is a quadruple consisting of the slope, the number of the tube, the quasi-socle and the quasi-length, the output are explicit matrices for the exceptional module with the data above. The basic tool for the algorithmic procedure is the following:

Theorem. *Let* Λ *be a tubular canonical algebra of type* $(p_1, \ldots, p_t)$ *and* E *a quasi-simple exceptional module in* $\mathrm{mod}_+(\Lambda)$ *with* $\mu(E) > p$ *and* $\mathrm{rk}(E) \geq 2$. *Then, with one exception in the tubular case* $(2, 3, 6)$*, there are exceptional* Λ*-modules* A *and* B *with* $\mu(A) \geq p$, $\mu(B) > p$ *and there is an exact sequence in* $\mathrm{mod}(\Lambda)$ $0 \to A \to E \to B \to 0$ *with* $\mathrm{dimExt}^1_\Lambda(B, A) = 1$.

Here rk, respectively μ, denotes the rank, respectively the slope, of a Λ-module, $\mathrm{mod}_+(\Lambda)$ denotes the full subcategory of $\mathrm{mod}(\Lambda)$ whose indecomposable objects are all Λ-modules of positive rank and $p = l.c.m.(p_1, \ldots, p_t)$. In the proof we use the fact that there is a weighted projective line $\mathbb{X}$ associated to Λ such that the category of finite dimensional modules over Λ and the category of coherent sheaves on $\mathbb{X}$ are derived equivalent [1]. We further describe a procedure to compute matrices for E provided matrices for A and B are known.

References

[1] W.Geigle and H.Lenzing *A class of weighted projective curves arising in representation theory of finite dimensional algebras.* In: Singularities, representations of algebras, and vector bundles. Springer Lecture Notes in Mathematics 1273 (1987), 265-297.

[2] H.Meltzer. *Exceptional modules for tubular canonical algebras.* Algebras and Representation Theory, to appear.

[3] C. M. Ringel. *Tame algebras and integral quadratic forms.* Springer Lecture Notes in Mathematics 1099 (1984).

83 Octavio Mendoza Hernandez

Universidad Nacional Autónoma de México, México City, México

Tilting categories

(joint with: *Corina Sàenz*)

In this talk we introduce the notion of Tilting Category. Our purpose is to get a good understanding of stratifying systems and their relationship with generalized tilting modules and some homological dimensions.

84 Héctor Merklen

Universidade de São Paulo, São Paulo, Brasil

On irreducible morphisms of complexes

(joint with: *Hermán Giraldo Salazar*)

Let $\mathcal{A}$ be an additive category (which, in most of the applications will be an abelian, Krull Schmidt, locally bounded $k-$category, for example, $\mathcal{A} = \Lambda-$mod, where Λ is a finite dimensional algebra) and let $\mathcal{P}$ be a full Krull-Schmidt, subcategory of $\mathcal{A}$.

C$\mathcal{A}$, (resp. C$\mathcal{P}$), denotes the category of unlimited complexes (to the left and to the right) $(X^\bullet, d^\bullet)$ of objects of $\mathcal{A}$, (resp. of $\mathcal{P}$).

In [1] it is shown that, if f is an irreducible morphism between complexes with an irreducible square, then there are exactly three possibilities:

1. all components of f are split monic;
2. all components of f are split epic;
3. there is exactly one component of f that is irreducible (in the category of the objects).

We characterize the irreducible morphisms of complexes which have a finite irreducible submorphism (not necessarily a square).

References

[1] Héctor Merklen and Hernán Giraldo Salazar, Irreducible morphisms of categories of complexes, submited for publication.

85 Vanessa Miemietz

Universität Köln, Köln, Germany

On the structure of Schur algebras for GL_2

(joint with: *Will Turner*)

We consider Ringel self-dual Schur algebras for $GL_2(F)$ and give a filtration by ideals, whose subquotients are isomorphic to smaller Ringel self-dual Schur algebras, their tilting modules and F-duals. We further construct two infinite-dimensional, self-injective, quasi-hereditary, Ringel self-dual algebras C and $\overline{C}$. These have subquotients isomorphic to the Schur algebra and its graded version (with respect to the filtration above) respectively. We show that C and $\overline{C}$ are stably equivalent.

86 Hiroki Miyahara

Shinshu University, Matsumoto, Japan

Homological properties over filtered rings

(joint with: *Kenji Nishida*)

Definition Let Λ be a (not necessarily commutative) ring. A family $\{\mathcal{F}_p\Lambda \mid p \in \mathbb{Z}\}$ of additive subgroups of Λ is called a *filtration* of Λ, if

(1) $\mathcal{F}_p\Lambda \subset \mathcal{F}_{p+1}\Lambda$ for all $p \in \mathbb{Z}$
(2) $\mathcal{F}_p\Lambda = 0$ for all $p < 0$
(3) $1 \in \mathcal{F}_0\Lambda$
(4) $(\mathcal{F}_p\Lambda)(\mathcal{F}_q\Lambda) \subset \mathcal{F}_{p+q}\Lambda$ for all $p, q \in \mathbb{Z}$
(5) $\Lambda = \bigcup_{p\in\mathbb{N}} \mathcal{F}_p\Lambda$

Λ is called a *filtered ring* if Λ has a filtration. If a ring Λ has a filtration $\mathcal{F}$, then $\bigoplus_{p\in\mathbb{N}} \mathcal{F}_p\Lambda/\mathcal{F}_{p-1}\Lambda$ is a graded ring with multiplication $\sigma_p(a)\sigma_q(b) = \sigma_{p+q}(ab)$ where $a \in \mathcal{F}_p\Lambda, b \in \mathcal{F}_q\Lambda$, and σ_p is a natural homomorphism from $\mathcal{F}_p\Lambda$ to $\mathcal{F}_p\Lambda/\mathcal{F}_{p-1}\Lambda$. We denote by $\mathrm{gr}\Lambda$ the above associated graded ring of Λ.

Let Λ be a filtered ring, and M a left Λ-module. A family $\{\mathcal{F}_p\Lambda \mid p \in \mathbb{Z}\}$ of additive subgroups of M is called a *filtration* of M, if

(1) $\mathcal{F}_pM \subset \mathcal{F}_{p+1}M$ for all $p \in \mathbb{Z}$
(2) $\mathcal{F}_pM = 0$ for $p \ll 0$
(3) $(\mathcal{F}_p\Lambda)(\mathcal{F}_qM) \subset \mathcal{F}_{p+q}M$ for all $p, q \in \mathbb{Z}$
(4) $M = \bigcup_{p\in\mathbb{Z}} \mathcal{F}_pM$

M is called a *filtered Λ-module* if M has a filtration. We denote by $\mathrm{gr}M$ the associated graded $\mathrm{gr}\Lambda$-module of M.

Homlogical properties over a filtered ring and its associated graded ring are investigated by J.-E. Björk, F. van Oystaeyen, et al. (see [1], [2]). We study the Gorenstein dimension and the grade of a finitely generated filtered module over a filtered ring. An equality or an inequality between these invariants of a filtered module and its associated graded module plays a very important role in the study of filtered rings. For example, J.-E. Björk proved in [1] that the grade of M is equal to the grade of $\mathrm{gr}M$ for every finitely generated filtered Λ-module M in case of grΛ is Auslander-Gorenstein. I will talk about some results which generalize those in [1] or [2].

References

[1] J.-E. Björk and E. K. Ekström,*Auslander Gorenstein rings*, in Operator algebras, unitary representations, enveloping algebras, and invariant theory, Progr. Math. 92, Birkhauser (1990) 425-448.

[2] Li Huishi and F. van Oystaeyen, *Zariskian Filtrations*, K-monographs in mathematics, 2, (1996)

87 Andrzej Mróz

Nicolaus Copernicus University, Toruń, Poland

An algorithmic approach to the multiplicity problem and to the isomorphism problem for representations of finite-dimensional algebras

(joint with: *Piotr Dowbor*)

We formulate the following two problems:

(a) *the multiplicity problem*, i.e. the problem of determining the multiplicities (up to isomorphism) of direct summands in "indecomposable decomposition" of a given Λ-module M, where Λ is a k-algebra,

(b) *the isomorphism problem*, i.e. the problem of deciding whether given two Λ-modules M, M' are isomorphic.

We discuss relationship between these two problems, a possibility of solving them algorithmically, and difficulties which appear.

In particular, we present the theorem on existence of an algorithm (up to finding the roots of polynomials) solving the multiplicity problem for domestic canonical algebras, with low polynomial computational complexity, and a fully verifiable criterion answering the question from the isomorphism problem, for this class of algebras.

References

1. P. Dowbor and A. Mróz, *The multiplicity problem for indecomposable decompositions of modules over a finite-dimensional algebra. Algorithms and a computer algebra approach*, Colloquium Math, 107 (2007), 221-261.

2. P. Dowbor and A. Mróz *The multiplicity problem for indecomposable decompositions of modules over domestic canonical algebras*, to appear in Colloquium Math.

3. P. Dowbor and A. Mróz, *On separating of orbits in the module variety for domestic canonical algebras*, to appear in Colloquium Math.

88 **Hiroshi Nagase**
Nara National College of Technology, Nara, Japan

Hochschild cohomology and stratifying ideals

(joint with: *Steffen König*)

When studying Hochschild cohomology it is natural to try relating cohomology of an algebra B to that of an 'easier' or 'smaller' algebra A. One such situation is that of B being a one-point extension of A, which has been studied by Happel. Happel's long exact sequence has been generalized to triangular matrix algebras independently by C. Cibils in [1] and by S. Michelena and M. I. Platzeck in [5] and to algebras with heredity ideals by J. A. de la Pena and C. C. Xi in [2].

We show that these results are generalized to the case of algebras with stratifying ideals. An idempotent ideal BeB of a finite dimensional algebra B is called *stratifying ideal* if the canonical algebra homomorphism $B \to B/BeB$ induces the fully faithful functor of the derived categories $D^+(\mathrm{mod}B/BeB) \to D^+(\mathrm{mod}B)$. When BeB is projective module as left (or right) B-module, we can show that BeB is stratifying ideal. So heredity ideals are examples of stratifying ideals and triangular matrix algebras have stratifying ideals. Our main result says that for any stratifying ideal BeB of a finite dimensional algebra B, there exist long exact sequences:
$\cdots \to \mathrm{Ext}^n_{B^e}(B, BeB) \to \mathrm{HH}^n(B) \to \mathrm{HH}^n(B/BeB) \to \cdots$; and
$\cdots \to \mathrm{Ext}^n_{B^e}(B/BeB, BeB) \to \mathrm{HH}^n(B) \to \mathrm{HH}^n(B/BeB) \oplus \mathrm{HH}^n(eBe) \to \cdots$.

The first long exact sequence has been already obtaind in the case that BeB is projective B-B-bimodule by J. A. de la Pena and C. C. Xi in [2].

We apply the second one to Hochschild cohomology rings modulo nilpotence. It is conjectured that Hochschild cohomology rings modulo nilpotence are finitely generated and, by E. L. Green, N. Snashall and O. Solberg, it has been shown that the conjecture is true in the case of monomial algebras, selfinjective algebras of finite representation type and so on in [4] and [3]. By using our long exact sequence, we give a way to make a series of algebras which satisfy the conjecture but are neither monomial nor selfinjective.

References

[1] C.Cibils: *Tensor Hochschild homology and cohomology*, in "Interaction between Ring Theory and Representations of Algebras," Lecture Notes in Applied Mathematics, Vol. 210, pp. 35–51, Dekker, NY, 2000.

[2] J. A. de la Pena and C. C. Xi: *Hochschild cohomology of algebras with homological ideals*, Tsukuba J. Math. 30 (2006), no. 1, 61-79.

[3] E. L. Green, N. Snashall and O. Solberg: *The Hochschild cohomology ring of a selfinjective algebra of finite representation type*, Proc. Amer. Math. Soc. 131 (2003), no. 11, 3387-3393.

[4] E. L. Green, N. Snashall and O. Solberg: *The Hochschild cohomology ring modulo nilpotence of a monomial algebra*, J. Algebra Appl. 5 (2006), no. 2, 153–192.

[5] S.Michelena and M.I.Platzeck: *Hochschild cohomology of triangular matrix algebras*, J. Algebra, **233** (2000), 502-525.

89 Gabriel Navarro

University of Granada, Granada, Spain

Localization in coalgebras and applications to their representation theory

In this talk we describe the theory of localization in categories of comodules. Path coalgebras are treated as a special example since, for this kind of coalgebras, localization is somehow combinatoric because of it is done by keeping and removing suitable group-like and skew-primitives elements. Several applications to Representation Theory of Coalgebras are shown. In particular we stand out from them some relations between the tameness and wildness of a coalgebra and its "localized" coalgebras, and also the following result: "Let K be an algebraically closed field and Q be an acyclic (probably infinite) quiver. Then any tame admissible subcoalgebra of KQ is the path coalgebra of a quiver with relations".

90 Quang Loc Nguyen

Nicolaus Copernicus University, Toruń, Poland

Regular orbit closures in module varieties

(joint with: *Grzegorz Zwara*)

Let A be a finitely generated associative algebra over an algebraically closed field. We characterize the finite dimensional A-modules whose orbit closures are regular varieties. Moreover, in case of characteristic zero, we describe the modules whose orbit closures are hypersurfaces.

91 Pedro Nicolás

Universidad de Murcia, Murcia, Spain

Parametrizing recollements of derived categories

The notion of *recollement* of a triangulated category was introduced by A. A. Beilinson, J. Bernstein and P. Deligne in their work on perverse sheaves. It consists in a way of seeing a triangulated category as glued together from two others triangulated categories. In general, recollements are in bijection with *TTF triples* (i.e. triples of subcategories formed by two concatenated pairs of t-structures), and with *smashing subcategories* when the ambient triangulated category is, for instance, compactly generated. We parametrize all the recollements of a compactly generated algebraic triangulated category by using a generalization of the notion of *homological epimorphism* due to W. Geigle and H. Lenzing. Our parametrization is somehow parallel to the corresponding one accomplished by J. P. Jans in 1965 on module categories.

92 Roland Olbricht

Bergische Universität Wuppertal, Wuppertal, Germany

Nori's variety and its singularities

The moduli space of representations of a fixed dimension $n \geq 2$ of the free associative algebra is a singular algebraic variety. M.V. Nori has developed an approach to desingularise it. For $n = 2$, Nori's construction yields indeed a desingularisation. We will describe its homology and the structure of the fibres of the desingularisation morphism. For $n > 2$, Nori's variety is no longer smooth. To understand its singularities, we study inside the variety of all n^2-dimensional algebras the orbit closure of the algebra of $n \times n$-matrices under the action of $GL(n^2)$. For $n = 3$ we will determine the singular locus of this orbit closure.

93 Steffen Oppermann

Universität Köln, Köln, Germany

Lower bounds for Auslander's representation dimension

The representation dimension is an invariant of a finite dimensional algebra introduced by Auslander, and expected to measure how far the algebra is from having finite representation type. However it seems to be difficult to establish lower bounds for the representation dimension, and it was not known, until Rouquier determined the representation dimension of exterior algebras, whether numbers greater than three can occur. I will suggest a way of extending Rouquier's definition of dimension of a triangulated category and indicate how this can help in finding lower bounds for the representation dimension of a given algebra. Then I will present a criterion which provides lower bounds for this new dimension, and therefore also for the representation dimension. Using this criterion, it will be possible to show that the representation dimension of $k[x_1, ..., x_n]/(x_1, ..., x_n)^n$ is $n + 1$, and to determine the representation dimensions of the members of related families of algebras of finite global dimension.

94 Rowena Paget

University of Kent, Canterbury, United Kingdom

Young modules for Brauer algebras

(joint with: *Robert Hartmann*)

Brauer algebras are diagram algebras which were introduced by Brauer in 1937. More recently, Graham and Lehrer showed that Brauer algebras (like group algebras of symmetric groups) are cellular algebras. Cell modules for the Brauer algebra play the role of Specht modules for the symmetric group. Here we define analogues for Brauer algebras of permutation modules and Young modules. We also obtain a Brauer algebra analogue of Hemmer and Nakano's theorem for symmetric groups. This is joint work with Robert Hartmann.

95 Yann Palu

Université Paris 7, Paris, France

Cluster characters for 2-Calabi–Yau triangulated categories

Starting from an arbitrary cluster-tilting object T in a 2-Calabi–Yau category $\mathcal{C}$ over an algebraically closed field, as in the setting of Keller and Reiten, we define, for each object L, a fraction $X(T,L)$ using a formula proposed by Caldero and Keller. We show that the map taking L to $X(T,L)$ is a cluster character, i.e. that it satisfies the following multiplication formula : If $\mathcal{C}(L,M)$ is one–dimensional, then

$$X(T,L)X(T,M) = X(T,B) + X(T,B')$$

where B and B' are given by two non-split triangles

$$L \to B \to M \to \Sigma L \text{ and } M \to B' \to L \to \Sigma M.$$

We deduce that it induces a bijection, in the finite and the acyclic case, between the indecomposable rigid objects of the cluster category and the cluster variables, which confirms a conjecture of Caldero and Keller.

96 Liangang Peng

Sichuan University, Chengdu, China

Conical extensions of derived categories

Our motivation is to construct certain new derived categories to approach toroidal Lie algebras via Ringel-Hall algebras. It is known that from the derived categories of representations of certain finite dimensional algebras, such as hereditary algebras and tubular algebras, one can approach Kac-Moody Lie algebras and elliptic Lie algebras. But it is hard to find a class of finite dimensional algebras such that their derived categories can be corresponding to general toroidal Lie algebras. In this talk, I will give an extension method to construct a derived category from two derived categories, called the conical extension. As an interest in Representation Theory of Algebra, we proved that if two given derived categories both have the Auslander-Reiten translations, then their conical extension derived category also has the Auslander-Reiten translation which is given explicitly. On the other hand, we also do some basic observations for relations between conical extension derived categories and toroidal Lie algebras.

97 Maria Julia Redondo

Universidad Nacional del Sur, Bahía Blanca, Argentina

Universal coefficient theorem in triangulated categories

(joint with: *Teimuraz Pirashvili*)

Let $h : \mathcal{T} \to \mathcal{A}$ be a homology theory on a triangulated category $\mathcal{T}$ with values in a graded abelian category $\mathcal{A}$. If the functor h reflects isomorphisms, is full and is such that for any object x in $\mathcal{A}$ there is an object X in $\mathcal{T}$ with an isomorphism between $h(X)$ and x, we prove that $\mathcal{A}$ is a hereditary abelian category, all idempotents in $\mathcal{T}$ split and the kernel of h is a square zero ideal which as a bifunctor on $\mathcal{T}$ is isomorphic to $\mathsf{Ext}^1_{\mathcal{A}}(h(-)[1], h(-))$.

98 Antonio Daniel Rivera López

Universidad Autónoma del Estado de Morelos, Cuernavaca, México

Generalized Serre relations for Lie algebras associated to positive quasi-Cartan matrix

Every semisimple Lie algebra defines a root system on the dual space of a Cartan subalgebra and a Cartan matrix, which expresses the dual of the Killing form on a root base. Serre's Theorem gives then a representation of the given Lie algebra by generators and relations in terms of the Cartan matrix. We want generalize Serre's Theorem to give an explicit representation by generators and relations for any semisimple Lie algebra in terms of a positive quasi-Cartan matrix.

99 Paweł Rochman

Nicolaus Copernicus University, Toruń, Poland

Classification of low-dimensional orbit closures in varieties of quiver representations

We classify the affine varieties of dimension not greater than four which occur as orbit closures of representations of quivers.

100 Wolfgang Rump

Universität Stuttgart, Stuttgart, Germany

L-functors and infinite rank representations

Let Λ be an isolated singularity in the sense of M. Auslander, i. e. Λ is an order over a complete regular local domain R with quotient field K, such that the category Λ-**CM** of Λ-modules which are finitely generated and free over R has almost split sequences. Then the homotopy category $\mathsf{M}(\Lambda\text{-}\mathbf{CM})$ of 2-termed complexes admits an adjoint pair $L \dashv L^-$ of L-functors, i. e. for an arbitrary morphism $f_0 \colon E_0 \to F_0$ in Λ-**CM**, the functor $L \colon \mathsf{M}(\Lambda\text{-}\mathbf{CM}) \to \mathsf{M}(\Lambda\text{-}\mathbf{CM})$ defines a ladder

$$\begin{array}{ccccccccccc} \cdots & \longrightarrow & E_4 & \longrightarrow & E_3 & \longrightarrow & E_2 & \longrightarrow & E_1 & \longrightarrow & E_0 \\ & & \big\downarrow f_4 & & \big\downarrow f_3 & & \big\downarrow f_2 & & \big\downarrow f_1 & & \big\downarrow f_0 \\ \cdots & \longrightarrow & F_4 & \longrightarrow & F_3 & \longrightarrow & F_2 & \longrightarrow & F_1 & \longrightarrow & F_0 \end{array}$$

with $f_n = L^n f_0$, and similarly, L^- defines a ladder in the opposite direction. The squares in the ladder give rise to short exact sequences

$$0 \to E_{n+1} \to E_n \oplus F_{n+1} \to F_n \to 0$$

closely related to almost split sequences.

In our talk, we give a brief introduction to L-functors and apply them to the construction of large representations of Λ. As a simplest example, ladders arise with any quasi-serial component in the Auslander-Reiten quiver of an artinian algebra. We generalize a result of Auslander, which states that an artinian algebra Λ has large indecomposables if Λ is not representation-finite, to arbitrary dimensions. If Λ is representation-finite, we show that the L^--ladder of the sink map $\vartheta\Lambda \to \Lambda$ contains a complete list of indecomposables as direct summands of the F_n. As a theoretical application, we use L-functors to show that the exact structure, or what is tantamount, the class of projective objects of Λ-**CM**, is determined by Λ-**CM** (as an additive category) if and only if the dimension of R is $\neq 2$.

101 Corina Sàenz

Universidad Nacional Autónoma de México, México City, México

Applications of tilting categories to stratifying systems

(joint with: *Octavio Mendoza*)

Let $(\theta, \leq)$ be a stratifying system of size t and $F(\theta)$ the category of all the R-modules that are filtered by modules in θ). Assuming that the category $I(\theta)$, of the relative Ext-injective R-modules of the category $F(\theta)$, is coresolving we prove that the category $F(\theta)$ is a partial tilting category. Moreover, using the theory developed for Tilting Categories, we get a theorem that generalizes some well known results concerning the finitistic dimension of properly stratified algebras.

102 Katsunori Sanada

Tokyo University of Science, Tokyo, Japan

On Hochschild cohomology of hereditary orders

(joint with: *Manabu Suda*)

We show that a hereditary order Λ over a complete discrete valuation ring with finite residue class field has a periodic projective resolution of period 2. We also compute the Hochschild cohomology ring of Λ by using the resolution. Therefore, the Hochschild cohomology ring of any hereditary order is also determined.

Let R be a complete discrete valuation ring with finite residue class field, and K be the quotient field of R. Let Λ be a basic hereditary R-order in a central simple K-algebra $A = M_m(D)$, where D is a division K-algebra with index n. We consider a periodic Λ^e-projective resolution of period 2 of Λ and the Hochschild cohomology $\mathrm{HH}^t(\Lambda) := \mathrm{Ext}^t_{\Lambda^e}(\Lambda, \Lambda)$ for $t \geq 0$, where Λ^e denotes the enveloping algebra of Λ.

The result for the case $n = 1$ or $m = 1$ is known. In this talk, we will describe that Λ has also a periodic Λ^e-projective resolution of period 2 in the general case $n, m \geq 1$ by generalizing the way of [Koenig, Snashall and Sanada; On Hochschild cohomology of orders, Arch. Math. **81** (2003), 627–635], and the Hochschild cohomology ring $\mathrm{HH}^*(\Lambda)$ is determined by means of the resolution and Yoneda product.

103 Selene Camelia Sanchez-Flores

Université de Montpellier 2, Montpellier, France

The Lie module structure on the Hochschild cohomology groups of monomial algebras of radical square zero

The Gerstenhaber bracket defined on the Hochschild cochains of an associative algebra provides the Hochschild cohomology with a Lie graded algebra structure. It follows that the first Hochschild cohomology group has a Lie algebra structure. Moreover, a Lie module structure over such Lie algebra is induced on the Hochschild cohomology groups. In this talk, we present results concerning the Lie module structure of the Hochschild cohomology groups of monomial algebras of radical square zero when the quiver is finite and connected. For such algebras, the Hochschild cohomology groups have been described by Cibils in terms of the combinatorics of the quiver. In order to compute the Gerstenhaber bracket using this description, we construct two quasi-isomorphisms between the Hochschild cochain complex and the complex induced by the reduced projective resolution. Once we have the calculations of the bracket, we go into the study of the Lie module structure when the quiver is given by two loops. In this case, we give a complete description of the Lie module structure of the Hochschild cohomology groups. Such description is based on combinatorial calculations and the classification of the irreducible Lie modules over simple Lie algebras when the ground field is the complex numbers.

104 Manuel Saorin

Universidad de Murcia, Murcia, Spain

Classification of compactly generated t-structures on the derived category of a Noetherian commutative ring

(joint with: *Leovigildo Alonso and Ana Jeremias*)

There are several examples in the literature where a finite dimensional A has a derived category $D(A) = D(A - Mod)$ which is equivalent to the derived category $D(X) = D(QcohX)$ of the category of quasi-coherent sheaves over some algebraic variety X. That gives an interesting bridge between Representation Theory and Algebraic Geometry. The notion of t-structure in a triangulated category, introduced by Beilinson, Bernstein and Deligne [*Faisceaux pervers.* Astrisque 100, Soc. Math. France, Paris 1982], is the correspondent in the triangulated context of the notion of torsion pair in an abelian category. If one wants to classify t-structures in $D(A)$, then it is natural to look at the corresponding problem on $D(X)$ and, afterwards, try to go back to $D(A)$. That makes the classification of t-structures in $D(X)$ an interesting problem for representation theorists.

In Algebraic Geometry one usually expects the affine case to be the easiest one, but even in this case no classification of t-structures in $D(X)$ is available. Even in that case, the general problem of classifying all the t-structures is hopeless. Indeed Stanley [*Invariants of t-structures and classification of nullity classes*, preprint: http://arxiv.org/abs/math/0602252] has shown that, when $X = Spec(\mathbf{Z})$ is the spectrum of the integers, the t-structures in $D(X)$ do not form a set. That shifts the target to more restrictive t-structures in $D(X)$, at least guaranteeing that they form a set. Obvious candidates are the t-structures in $D(X)$ which are generated by significative sets of

objects (e.g. compact objects, objects of bounded and coherent cohomology, etc.).

In our work we pursue the last line of action, in the case of X being an affine Noetherian scheme. Therefore $X = Spec(R)$, for a commutative Noetherian ring R, and $D(X)$ is equivalent to $D(R) = D(R - Mod)$. We fix our notation and terminology. Given a superindex $*$ in the set $\{b, -, +, 'blank'\}$ indicating the boundedness on the homology, we will denote by $D^*_{fg}(R)$ the full (triangulated) subcategory of $D^*(R)$ formed by the objects having finitely generated homology in each degree. A map $\phi : \mathbf{Z} \longrightarrow \mathcal{P}(Spec(R))$ is called a *filtration by supports* in $Spec(R)$ in case it is decreasing and each $\phi(i)$ is a stable under specialization subset of $Spec(R)$. Finally, recall that if $(\mathcal{U}, \mathcal{U}^{\perp}[1])$ is a t-structure in $D(R)$, then $\mathcal{U}$ is called the *aisle* of $D(R)$. That aisle (or the t-structure) is generated by a subset $\mathcal{S} \subseteq \mathcal{U}$ in case $\mathcal{U}$ is the smallest aisle in $D(R)$ containing $\mathcal{S}$.

Our main result is the following:

THEOREM 1.- Let R be a Noetherian commutative ring and $(\mathcal{U}, \mathcal{U}^{\perp}[1])$ be a t-structure in $D(R)$. The following assertions are equivalent:

1. *$\mathcal{U}$ is compactly generated*
2. *$\mathcal{U}$ is generated by stalk complexes of finitely generated (resp. cyclic) R-modules*
3. *$\mathcal{U}$ is generated by complexes of $D^-_{fg}(R)$*
4. *There is a (uniquely determined) filtration by supports $\phi : \mathbf{Z} \longrightarrow \mathcal{P}(Spec(R))$ such that $\mathcal{U}$ is the aisle in $D(R)$ generated by $\{R/\mathbf{p}[-i] : i \in \mathbf{Z} \text{ and } \mathbf{p} \in \phi(i)\}$*

As a consequence we obtain a classification of the compactly generated t-structures of $D(R)$ in the spirit of Neeman's classification of triangulated subcategories [*The chromatic tower for D(R)*. Topology 31 (1992), 519-532]. Namely, we get:

COROLLARY.- Under the hypotheses of the above theorem, there is a one-to-one correspondence between:

1. *Filtrations by supports of $Spec(R)$*
2. *Compactly generated t-structures in $D(R)$*

The correspondence maps the t-structure $(\mathcal{U}, \mathcal{U}^{\perp}[1])$ onto $\phi := \phi_{\mathcal{U}} : \mathbf{Z} \longrightarrow \mathcal{P}(Spec(R))$, defined by $\phi(i) = \{\mathbf{p} \in Spec(R) : R/\mathbf{p}[-i] \in \mathcal{U}\}$. Conversely, it maps the filtration by supports ϕ onto the t-structure $(\mathcal{U}_{\phi}, \mathcal{U}_{\phi}^{\perp}[1])$ whose aisle is explicitly defined as:

$$\mathcal{U}_{\phi} = \{M^{\cdot} \in D(R) : Supp(H^i(M^{\cdot})) \subseteq \phi(i), \text{ for all } i \in \mathbf{Z}\}$$

Another consequence of the main theorem is that every t-structure in $D^b_{fg}(R)$ is the restriction of a unique compactly generated t-structure in $D(R)$. Then the following three problems are equivalent:

1. Classifying the t-structures of $D^b_{fg}(R)$
2. Classifying the compactly generated t-structures of $D(R)$ which restrict to t-structures in $D^b_{fg}(R)$

3. Identifying the filtrations by supports ϕ in $Spec(R)$ such that the associated t-structure $(\mathcal{U}_\phi, \mathcal{U}_\phi^\perp[1])$ restricts to $D^b_{fg}(R)$

In case of having time, we shall discuss the third of these equivalent problems and give a quite general solution:

THEOREM 2.- Let R be a Noetherian commutative ring and $\phi : \mathbf{Z} \longrightarrow \mathcal{P}(Spec(R))$ *be a filtration by supports. Consider the following assertions:*

1. *The associated t-structure* $(\mathcal{U}_\phi, \mathcal{U}_\phi^\perp[1])$ *restricts to* $D^b_{fg}(R)$
2. ϕ *satisfies the weak Cousin condition (i.e. if* $i \in \mathbf{Z}$ *is an integer and* $\mathbf{p} \subsetneq \mathbf{q}$ *are prime ideals such that* $\mathbf{p}$ *is maximal under* $\mathbf{q}$ *and* $\mathbf{q} \in \phi(i)$*, then* $\mathbf{p} \in \phi(i-1)$*)*

Then (1) $\Longrightarrow$ (2) *and, in case R has a dualizing complex (e.g. if R is a finitely generated algebra over a field or over a PID), the reverse implication also holds.*

105 Sarah Scherotzke

Mathematical Institute Oxford, Oxford, United Kingdom

Classification of pointed rank one Hopf algebras

We classify pointed rank one Hopf algebras over fields of prime characteristic which are generated as algebras by the first term of the coradical filtration. In the case of characteristic zero this classification has been done by Radford and Krop. We obtain three types of Hopf algebras presented by generators and relations. In characteristic zero only one of the three types occur. For Hopf algebras with semi-simple coradical only the first and second type appears. We determine the indecomposable projective modules for certain classes of pointed rank one Hopf algebras.

106 Ralf Schiffler

University of Massachusetts Amherst, Amherst, USA

Geometric realizations of cluster categories

The cluster category of a hereditary algebra is usually defined as a quotient of the bounded derived category of modules of that algebra. In this talk, we will consider an alternative definition of the cluster categories of Dynkin types A and D. In type A, the cluster category is a category of diagonals in a regular polygon, and, in type D, the cluster category is a category of certain homotopy classes of paths in a regular polygon with one puncture in its center.

107 Markus Schmidmeier

Florida Atlantic University, Boca Raton, USA

Nilpotent linear operators and their endostructure

We consider triples $X = (V, U, T)$ consisting of a finite dimensional vector space V, a nilpotent linear operator $T : V \to V$ and a subspace U of V which is invariant under the action of T. Denote by $S(n)$ the category of those triples where the operator acts with nilpotency index at most n.

The case where $n = 6$ is of particular interest as $S(n)$ has finite type if $n < 6$ and is wild if $n > 6$. It has been shown by D. Simson that the category $S(6)$ is tame of polynomial growth.

My talk is a report on joint work with C. M. Ringel on the structure of objects in $S(6)$. The infinite ideal I in $S(6)$ is an idempotent ideal so $I^2 = I$ holds. Still if X is an indecomposable object, then the ideal $I(X, X)$ of $\mathrm{End}(X)$ has nilpotency index at most 8.

The factor $\mathrm{End}(X)/I(X, X)$ is isomorphic to a subring S of $\mathrm{End}(X)$. It turns out that whenever the dimension type $(\dim V, \dim U)$ of X is an integer multiple of (12,6), then X is free as an S-module. Conversely if S is not a field and X is free as an S-module, then the dimension type of X is a multiple of (12,6).

108 Karsten Schmidt

Universität Paderborn, Paderborn, Germany

The number of Auslander-Reiten components for simply connected differential graded algebras

Peter Jørgensen has introduced the Auslander-Reiten quiver of a simply connected Poincaré duality space. He shows that its components are of the form $\mathbb{Z}A_\infty$ and that the Auslander-Reiten quiver of a d-dimensional sphere consists of $d-1$ such components. We will show that the example of the spheres is the only case where finitely many components appear. More precisely, depending on the cohomology dimensions of the appearing differential graded algebras, we construct discrete and continuous families of modules lying in different components.

109 Claudio Schmidt-Wegenast

Heinrich-Heine-Universität Düsseldorf, Düsseldorf, Germany

Endomorphism rings of regular modules over wild hereditary algebras

Let H be a connected wild hereditary path-algebra over some algebraically closed field k. For a quasi-simple regular brick Z in H-mod let be $t = \min\{l \in \mathbb{N} \mid \mathrm{Ext}^1_H([l]Z, [l]Z) \neq 0\}$ where $[m]Z$ is the indecomposable regular module of quasi-lenght m and quasi-socle Z. Then $\lceil \frac{m}{t} \rceil$ is an upper bound for the degree of nilpotency of $\mathrm{radEnd}_H([m]Z)$. This bound is sharp if Z is elementary.

For elemantary modules E and $m \geq t+1$ we additionally have that $\mathrm{End}_H([m-t]E)$ is isomorphic to the factor ring $\mathrm{End}_H([m]E)/S$ where S is the left socle respectively right socle of $\mathrm{End}_H([m]E)$.

110 Sibylle Schroll

University of Oxford, Oxford, United Kingdom

On rhombal algebras and rhombus filtrations

(joint with: *Alex Clark and Karin Erdmann*)

We will illustrate on a particular family of rhombal algebras, the so-called Rauzy algebras, how *rhombus filtrations* can be used to determine part of the stable Auslander-Reiten structure of these algebras.

111 Jan Schröer

Universität Bonn, Bonn, Germany

Mutations of multisegments and applications to cluster algebras

(joint with: *Christof Geiss and Bernard Leclerc*)

Let Q be a finite quiver without oriented cycles, and let M be a preinjective KQ-module such that add(M) is closed under factor modules.

Using combinatorics in the spirit of Auslander-Reiten theory, we will define the mutation of certain tuples $(X_1, ..., X_r)$, called "multisegments", where the X_i are modules in add(M).

This yields a new combinatorial model describing many cluster algebras, including all acyclic cluster algebras and also many cluster algebras arising from Lie theory.

Our model yields a conjecture about the existence of a "canonical basis" for these cluster algebras, where the basis vectors are parametrized by the modules in add(M).

If there is enough time, we will explain how our results relate to the partial ordering on the set of tilting modules over certain quasi-hereditary algebras.

112 Jeanne Scott

University of Leeds, Leeds, United Kingdom

The affine GLS φ-map and 'chess' Kostka numbers

(related to work with: *Aslak Bakke Buan, Osamu Iyama and Idun Reiten*)

I will discuss how to compute the Geiss-Leclerc-Schröer φ-map for "shape modules" over the affine preprojective algebra in terms of 'chess' Kostka numbers.

113 Maja Sędłak

Nicolaus Copernicus University, Toruń, Poland

On the question if representation-finite algebras form an open $\mathbb{Z}$-scheme

The question if representation-finite algebras form an open $\mathbb{Z}$-scheme, formulated by Ch.U. Jensen and H. Lenzing, can be reduced by van den Dries's test to the following problem:

Let V be a valuation ring in an algebraically field K with the residue field R. Given a V-order A we denote $\overline{A} = A \otimes_V R$, $A^{(K)} = A \otimes_V K$. Is it true that the K-algebra $A^{(K)}$ is representation-finite provided the R-algebra $\overline{A}$ is representation-finite?
The answer is not known.

We define a class of quivers called tree extensions of a cycle. A finite quiver Q is a tree extension of a cycle if the fundamental group of Q is infinite cyclic and Q contains (exactly one) oriented cycle Δ such that there is no oriented path in Q starting at the cycle Δ and ending outside Δ. The main aim of the talk is to present the following theorem:

THEOREM. *If $\overline{A}$ is representation-finite algebra and the Gabriel quiver of $\overline{A}$ is a tree extension of a cycle then the algebra $A^{(K)}$ is representation-finite.*

114 Chelliah Selvaraj

Periyar University, Salem, India

Factor algebras of signed Brauer's algebras

(joint with: *Nil*)

In this paper we obtain a decomposition of certain factors of the signed Brauer algebra into a direct sum of simple algebras and we obtain the structure of the factor algebra.

115 Ahmet Seven

Middle East Technical University, Ankara, Turkey

Mutation classes of quivers and quasi-Cartan matrices

Quasi-Cartan matrices, defined by Barot-Geiss-Zelevinsky, are analogues of generalized Cartan matrices for quivers which are not necessarily acyclic. In this talk, we will discuss quasi-Cartan matrices of some quivers and describe their relation to mutation classes.

116 Sverre O. Smalø

Norwegian University of Science and Technology, Trondheim, Norway

Degenerations and virtual degenerations are usually not the same.

(joint with: *Christine Riedtmann*)

Since the first example of John Carlson, there has not been given to many examples where the degeneration order and the virtual degeneration order is not the same. Here, several examples where this happens will be given, including a proof that for wild hereditary algebras these two partial orders are not the same except for some few small dimension vectors.

117 David Smith

Norwegian University of Science and Technology, Trondheim, Norway

On special tilting modules over cluster-tilted algebras

The cluster-tilted algebras of Buan, Marsh and Reiten turned out to be very important in representation theory of algebras. An interesting problem is then to determine which algebras are derived-equivalent to cluster-tilted algebras, and thus to understand the tilting theory over cluster-tilted algebras. Up to now, very few is known about the endomorphism rings of tilting modules over cluster-tilted algebras. Easy examples show that they are generally not cluster-tilted. In this talk, we use the left and right supported property of cluster-tilted algebras to determine a finite set of tilting modules whose endomorphism rings are still cluster-tilted. Moreover, we show that these can be obtained from the original one by performing a finite number of Bernstein-Gelfand-Ponomarev reflections.

118 Nicole Snashall

University of Leicester, Leicester, United Kingdom

The Hochschild cohomology ring modulo nilpotence

For a finite-dimensional algebra Λ, it was conjectured by Snashall and Solberg that the Hochschild cohomology ring of Λ modulo nilpotence is itself a finitely generated algebra (Proc. LMS 2004). This talk will give a brief survey of the current position of this conjecture and its connection to support varieties of modules. In particular, I will discuss the cases when Λ is a monomial algebra (Green, Snashall, Solberg, J. Alg. and Appl. 2006) or is in a class of special biserial algebras which arise from the representation theory of $U_q(\mathfrak{sl}_2)$ (Erdmann, Snashall, Taillefer).

119 Øyvind Solberg

Norwegian University of Science and Technology, Trondheim, Norway

Koszul theory for all finite dimensional algebras

(joint with: *Roberto Martinez Villa*)

Let Λ be a finite dimensional algebra over a field k. Our final aim is to explain how one can associate a Frobenius Koszul category with radical cube zero to any connected regular component of the Auslander-Reiten quiver of Λ. To do this we introduce Koszul categories, and we show there is a Koszul duality between a Koszul category and its Koszul dual category.

120 María José Souto Salorio

Universidade da Coruña, Coruña, Spain

Aisles and silting complexes in $\mathbf{D^b}(\mathcal{H})$

(joint with: *Ibrahim Assem and Sonia Trepode*)

Let $\mathcal{H}$ be a hereditary abelian k-category with tilting object and $\mathbf{D^b}(\mathcal{H})$ denote the bounded derived category of $\mathcal{H}$. We study suspended subcategories and t-structures on $\mathbf{D^b}(\mathcal{H})$ by means of their Ext-projectives and silting complexes. In particular, we focus our attention on the category $\mathcal{H} = \operatorname{mod} A$ where A is the Kronecker algebra.

References

[H] Happel, D., *Triangulated categories in the representation theory of finite-dimensional algebras*, London Math. Soc. Lecture Notes Series **119**, Cambridge University Press, 1988.

[HR] Happel, D., Reiten, I., *Hereditary categories with tilting object*, Math. Z. **232** (1999), 559-588.

[HRS] Happel, D., Reiten, I., Smalø, S. O., *Tilting in abelian categories and quasitilted algebras*, Mem. Amer. Math. Soc. 120 (1996), **575**, 88 pp.

[KV] Keller B., Vossieck D., *Aisles in derived categories*, Bull. Soc. Math. Belg. **40, (2)** (1988), 239-253.

[RV] Reiten, I., Van der Bergh, M., *Grothendieck groups and tilting objects*, Algebras and Representations Theory **4** (2001) 257-272.

[Ri] Ringel, C. M., *Tame algebras and integral quadratic forms*, Springer Lect. Notes Math. **1099** (1984).

121 Jan Stovicek

Norwegian University of Science and Technology, Trondheim, Norway

Idempotent ideals in a module category and the Telescope Conjecture

Let $\mathcal{C}$ be a Hom-finite additive category with splitting idempotents. I will discuss the structure of (two-sided) idempotent ideals of $\mathcal{C}$. I will show that this depends very much on the structure of the transfinite radical of $\mathcal{C}$; that is, the unique maximal idempotent ideal inside the radical of $\mathcal{C}$. If $\mathcal{C} = \mathrm{mod}\Lambda$ for a finite dimensional algebra Λ and the transfinite radical of $\mathrm{mod}\Lambda$ vanishes, then the following version of the Telescope Conjecture for Module Categories holds: If $\mathcal{B} \subseteq \mathrm{Mod}\Lambda$ is a class of modules closed under taking direct limits and $\mathcal{B} = \mathrm{Ker}\,\mathrm{Ext}^1_\Lambda(\mathcal{S}, -)$ for some class $\mathcal{S}$, then we can take $\mathcal{S}$ to be a set of finitely generated modules. Algebras with zero transfinite radical include for example domestic standard selfinjective algebras and domestic special biserial algebras. This talk is partly based on a joint work with Jan Šaroch.

122 Xiuping Su

Universität Köln, Köln, Germany

Exceptional representations of a double quiver of type A, and Richardson elements in seaweed Lie algebras

(joint with: *Bernt Tore Jensen and Rupert W. T. Yu*)

Let $\mathfrak{g}$ be a reductive Lie algebra over an algebraically closed field k. A Lie subalgebra $\mathfrak{q}$ of $\mathfrak{g}$ is called a seaweed Lie algebra if there exists a pair of parabolic subalgebras $(\mathfrak{p}, \mathfrak{p}')$ of $\mathfrak{g}$ such that $\mathfrak{q} = \mathfrak{p} \cap \mathfrak{p}'$ and $\mathfrak{p} + \mathfrak{p}' = \mathfrak{g}$. Seaweed Lie algebras are introduced by Vladimir Dergachev and Alexander Kirillov in the case $\mathfrak{g} = \mathrm{gl}_n(k)$, and in the general case by Dmitri Panyushev. The set of seaweed Lie algebras in $\mathfrak{g}$ contains clearly all parabolic subalgebras and Levi factors of $\mathfrak{g}$. In particular, they provide new examples of index zero Lie algebras (or Frobenius Lie algebras). We denote by Q the corresponding Lie group of $\mathfrak{q}$. We are interested in the following question raised independently by Michel Duflo and Dmitri Panyushev:

Question 1 Is there an open Q-orbit in the nilpotent radical $\mathfrak{n}$ of $\mathfrak{q}$?

In the case where $\mathfrak{q}$ is a parabolic subalgebra of $\mathfrak{g}$, the answer is yes and there is a unique open Q-orbit. This is a result that is commonly known as Richardson's Dense Orbit Theorem. If there is an open Q-orbit in the nilpotent radical of $\mathfrak{q}$, then an element in the open Q-orbit is called a Richardson element of $\mathfrak{q}$. Yu found a seaweed Lie algebra in a Lie algebra of type E_8 whose nilpotent radical does not contain an open Q-orbit.

Our present task is to study the case of seaweed Lie algebras in $\mathrm{gl}_n(k)$. In this particular case, seaweed Lie algebras can be viewed as the stabiliser of a pair of weakly opposite flags. This provides a nice description of seaweed Lie algebras, and allows us to transfer the problem to a quiver representations setting, generalising the one for parabolic subalgebras considered by Thomas Brüstle, Lutz Hille, Claus Ringel and Gerhard Röhrle. More precisely, we associate to $\mathfrak{q}$ a certain double quiver $(\tilde{Q}, \mathcal{I})$ of type A together with a dimension vector d, where $\mathcal{I}$ is relations determined by $\mathfrak{q}$. Then the quotient of the path algebra by the relations $\mathcal{I}$ has a structure of a quasi-hereditary algebra with respect to $(\{\Delta(i)\}_i, \leq)$, where $\Delta(i)$ is a fixed representation for each vertex i and $\leq$ is a partial order on the set of vertices of Q. These quotient algebras have been studied by several people, for example Steffen König, Changchang Xi, Lutz Hille, etc.

We denote by $\underline{\dim}(M)$ the dimension vector of a representation M. We say that a Δ-filtered representation M has Δ-dimension vector d if the dimension vector $\underline{\dim}(M)$ of M can be written as a sum $\sum_i d_i \underline{\dim}(\Delta(i))$. We denote by $Rep_\Delta(\tilde{Q}, \mathcal{I}, d)$ the representation variety of Δ-filtered representations with Δ-dimension vector d. Our main results are:

Theorem 2 Given a Δ-dimension vector d, there exists a unique (up to isomorphism) exceptional Δ-filtered module with Δ-dimension vector d.

Theorem 3 There is a one-to-one correspondence between the set of Q-orbits in the nilpotent radical of $\mathfrak{q}$ and the set of $\mathrm{GL}(d)$-orbits in the representation variety $Rep_\Delta(\tilde{Q}, \mathcal{I}, d)$. Moreover a Q-orbit is open in $\mathfrak{q}$ if and only if the corresponding $\mathrm{GL}(d)$-orbit is open in $Rep_\Delta(\tilde{Q}, \mathcal{I}, d)$.

As a corollary of the above two theorems we have a positive answer to Question 1 for seaweed Lie algebras in $\mathrm{gl}_n(k)$

Theorem 4 For any seaweed Lie algebra $\mathfrak{q}$ in $\mathrm{gl}_n(k)$ there is a unique open Q-orbit in the nilpotent radical of q.

123 Csaba Szántó

"Babeş-Bolyai" University Cluj-Napoca, Cluj-Napoca, Romania

Representation theoretical methods in matrix pencil theory

It is well known that matrix pencils correspond to Kronecker modules. Using the modular approach and representation theoretical techniques it is possible to obtain new results for computing the Kronecker invariants and for counting the (co)dimension of orbits. Using a Hall algebra approach (joint with A. Hubery) we can characterize in terms of Kronecker invariants when a given pencil is a subpencil of an another one.

124 Karolina Szmyt

Nicolaus Copernicus University, Toruń, Poland

Tilting slice modules over minimal 2-fundamental algebras

(joint with: *Zygmunt Pogorzały*)

My talk is a report on a joint work with Zygmunt Pogorzaly. We consider minimal 2-fundamental algebras whose Auslander-Reiten quiver contains a component at the beginning that is not generalized standard and contains the projective vertices. For these components we introduce a generalization of a slice. We have already known that there are only finitely many postprojective slices S whose slice modules M_S are tilting modules. We present a detailed description of the postprojective slices S whose slice modules M_S are tilting modules. Our main results are the follwing:

Theorem 1. *For a minimal* 2*-fundamental algebra* A *let* $\mathcal{C}$ *be the starting component in* Γ_A *that is not generalized standard. Let* $S=\{X_i\}_{i=1}^t$ *be a postprojective slice in* $\mathcal{C}$*. Then* *the slice module $X_S = \bigoplus_{i=1}^{t} X_i$ is a tilting A-module if and only if S is contained in the postprojective starting cone C_{SC} in $\mathcal{C}$.*

Theorem 2. *For a minimal 2-fundamental algebra A let $\mathcal{C}$ be the starting component in Γ_A that is not generalized standard. Let S be a postprojective slice in $\mathcal{C}$ that is contained in the postprojective starting cone C_{SC} in $\mathcal{C}$. Then the algebra $End_A(X_S)^{op}$ is a (t,p,s)-algebra.*

125 Hermund Andre Torkildsen

Norwegian University of Science and Technology, Trondheim, Norway

Counting cluster-tilted algebras of finite representation type

Cluster categories were introduced in [BMRRT] and independently in [CCS1] for the A_n case. Let $\mathcal{C}$ be a cluster category. Then a cluster-tilted algebra is $\mathrm{End}_{\mathcal{C}}(T)^{\mathrm{op}}$, where T is a cluster-tilting object in $\mathcal{C}$. It is known that the number of non-isomorphic cluster-tilted algebras of finite representation type is finite. The number of cluster-tilted algebras, up to isomorphism, of a fixed Dynkin type $\vec{\Delta}$ is equal to the size of the mutation class of Δ, i.e. the number of quivers, up to isomorphism, obtained from $\vec{\Delta}$ by repeated mutation. We will give the number of isomorphism classes of cluster-tilted algebras of type A_n and D_n explicitly.

From [CCS1], it is known that for any triangulation $\mathcal{T}$ of a regular $(n+3)$-polygon, we can define a quiver $Q_{\mathcal{T}}$. This quiver is a quiver of a cluster-tilted algebra of type A_n. We will see that the number of non-isomorphic cluster-tilted algebras of type A_n is the number of triangulations of a regular $(n+3)$-polygon up to rotation, i.e. the number of triangulations of the disk. These numbers were given in [B]. From this it will also follow that if T and T' are cluster-tilting objects in $\mathcal{C}$, then $T = \tau^i T'$ if and only if $\mathrm{End}_{\mathcal{C}}(T)$ is isomorphic to $\mathrm{End}_{\mathcal{C}}(T')$.

We also discuss the D_n case.

References

[B] Brown, W. G. *Enumeration of triangulations of the disk*, Proc. London Math. Soc., 14 (1964), 746-768.

[BMR] Buan A., Marsh R., Reiten I. *Cluster-tilted algebras*, Trans. Amer. Math. Soc., 359, no. 1, 323–332 (2007).

[BMRRT] Buan A., Marsh R., Reineke M., Reiten I., Todorov G. *Tilting theory and cluster combinatorics*, Advances in mathematics, 204 (2), 572-618 (2006).

[CCS1] Caldero P., Chapton F., Schiffler R. *Quivers with relations arising from clusters (A_n case)*, Trans. Amer. Math. Soc. 358 , no. 3, (2006) 1347-1364.

126 Sonia Trepode

Universidad Nacional de Mar del Plata, Mar del Plata, Argentina

Contravariantly finite subcategories closed under predecessors

(joint with: *Ibrahim Assem and Flavio Coelho*)

In this talk we consider a full subcategory C of mod A closed under predecessors. We characterise when add C is contravariantly finite. In particular we show that the fact of add C be contravariantly finite is associated with the existence of a cotilting module over an appropiated algebra. We obtain that in several situations this cotilting module "splits" the module category in two parts. We apply these results to triseccions of the module category.

127 Will Turner

University of Oxford, Oxford, United Kingdom

Homomorphic symplectic algebras

In recent years, some energy has been devoted to placing a geometric restriction, the Calabi-Yau property, in a noncommutative algebraic setting. The category of holomorphic symplectic manifolds forms an interesting subcategory of the category of Calabi-Yau manifolds. Here, we discuss an algebraic restriction, defined in homological terms, which appears to be symplectic in nature. We discuss the Cubist algebras, which form a rich family of examples, and are relevant in the modular representation theory of finite groups.

128 Helene Tyler

Manhattan College, New York, USA

Sequences of reflection functors and the preprojective component of a valued quiver

This talk concerns preprojective representations of a finite connected valued quiver without oriented cycles. For each such representation, an explicit formula in terms of the geometry of the quiver gives a unique, up to a certain equivalence, shortest (+)-admissible sequence such that the corresponding composition of reflection functors annihilates the representation. The set of equivalence classes of the above sequences is a partially ordered set that contains a great deal of information about the preprojective component of the Auslander-Reiten quiver. The results apply to the study of reduced words in the Weyl group associated to an indecomposable symmetrizable generalized Cartan matrix.

129 Geert Van de Weyer

University of Antwerp, Antwerp, Belgium

Linear double Poisson algebras

(joint with: *Anne Pichereau*)

We give a description of the graded Lie algebra of noncommutative poly-vectorfields of the path algebra of a quiver in terms of a graded necklace algebra of a quiver. We use this description to classify linear double Poisson structures on a quiver setting. For the free algebra F in d variables over an algebraically closed field of characteristic zero, we show there is a one-to-one correspondence between linear double Poisson brackets on F and finite dimensional algebra structures on a d-dimensional vector space. Moreover, we show the Hochschild cohomology of this algebra structure can be recovered from the double Poisson cohomology of the corresponding double Poisson bracket.

130 Dagfinn Vatne

Norwegian University of Science and Technology, Trondheim, Norway

Cluster-tilted algebras of type A

(joint with: *Aslak Bakke Buan*)

We will classify all cluster-tilted algebras of type A_n. All of these algebras turn out to be gentle. We use this classification to decide exactly when two such algebras are derived equivalent: This happens if and only if their quivers have the same number of 3-cycles.

Reference: arXiv:math.RT/0701612v1

131 Takayoshi Wakamatsu

Saitama University, Saitama, Japan

On Nakayama automorphisms of Frobenius graded algebras

Let K be an algebraically closed field. Any Frobenius local graded K-algebra Λ can be constructed from a multilinear map $\varphi : V^{\otimes n} \to K$ satisfying the condition

$$\varphi(x \otimes w) = \varphi(w \otimes \gamma(x))$$

where V is a finite-dimensional K-vector space, $\gamma \in \mathrm{GL}(V)$ and $x \in V$, $w \in V^{\otimes(n-1)}$ are arbitrary elements, by setting $\Lambda_k = V^{\otimes k}/\mathrm{Ker}(\varphi_k)$ for all $0 \le k \le n$ and

$$\Lambda = \Lambda_0 \oplus \Lambda_1 \oplus \cdots\cdots \oplus \Lambda_{n-1} \oplus \Lambda_n.$$

Here, the multilinear map $\varphi_k : V^{\otimes k} \to \mathrm{Hom}_K(V^{\otimes(n-k)}, K)$ is defined by $\varphi_k(w)(z) = \varphi(w \otimes z)$ for elements $w \in V^{\otimes k}$ and $z \in V^{\otimes(n-k)}$. By using this construction of Frobenius algebras, we can study Nakayama automorphisms of Frobenius algebras.

132 Torsten Wedhorn

Universität Paderborn, Paderborn, Germany

Invariants in arithmetic geometry, clan representations, and the wonderful compactification

To several key objects in arithmetic geometry (varieties over fields of positive characteristic, p-adic representations of Galois groups, points of certain Shimura varieties) it is possible to associate via cohomology an object which is called an F-zip. These objects can, on one hand, be considered as (a semi-linear variant of) a representation of a certain clan in the sense of Crawley-Boevey. On the other hand, F-zips form a variety which is closely connected to certain pieces in the wonderful compactification of $PGL(n)$ which have been defined by Lusztig in his theory of parabolic character sheaves.

In this talk I will try to tie these three worlds together concentrating on applications of results of Crawley-Boevey and Geis for clan representations in arithmetic geometry. At the end I will ask some questions which come up naturally in arithmetic geometry but which may be new in representation theory.

133 Michael Wemyss

University of Bristol, Bristol, United Kingdom

Reconstruction algebras and non-commutative geometry

For G a finite subgroup of $SL(2)$ the classical McKay correspondence relates the minimal resolution of the singularity $\mathbb{C}^2/G$ to the representation theory of G. All information about the geometry is encoded by the skew group ring (which is non-commutative) which we can view as the preprojective algebra of an extended Dynkin diagram. For finite cyclic subgroups of $GL(2)$ I'll show how to generalise this notion to that of a reconstruction algebra and then show how to obtain all the geometry from this new non-commutative ring

134 Stefan Wolf

Universität Paderborn, Paderborn, Germany

The composition monoid and the composition algebra at $q = 0$

Let Q be a Dynkin quiver or an equioriented cyclic quiver and let modkQ be the category of nilpotent finite dimensional kQ modules for k a field. Then for each set of modules $A, B, M \in \text{mod}kQ$ there are polynomials $f_{AB}^M(q) \in \mathbb{Z}[q]$ such that $F_{AB}^M = f_{AB}^M(|k|)$, where F_{AB}^M are the Hall numbers. By using these polynomials as structure coefficients one defines the generic Ringel-Hall algebra $\mathcal{H}_q(Q)$. The composition algebra $\mathcal{C}_q(Q)$ is the subalgebra of the Ringel-Hall algebra generated by the simple modules.

Now let k be an algebraically closed field. For $\alpha \in \mathbb{N}^{Q_0}$ let $\text{Rep}_k(\alpha) := \bigoplus_{\substack{a:i\to j\\ a\in Q_1}} k^{\alpha_j \times \alpha_i}$ be the representation variety. Let $\mathcal{A}, \mathcal{B}$ be closed, irreducible, $\text{Gl}_\alpha(k)$ resp. $\text{Gl}_\beta(k)$ stable subvarieties of $\mathrm{Rep}_k(\alpha)$ resp. $\mathrm{Rep}_k(\beta)$. One defines

$$\mathcal{A} * \mathcal{B} := \{M \in \mathrm{Rep}(\alpha + \beta) : \exists A \in \mathcal{A}, B \in \mathcal{B}, 0 \to B \to M \to A \to 0 \text{ exact}\}$$

This is again closed, irreducible, $\mathrm{Gl}_{\alpha+\beta}(k)$ stable. The multiplication is associative and we obtain a monoid $\mathcal{M}_k(Q)$. The composition monoid $\mathcal{CM}(Q)$ is the submonoid generated by the simple modules.

Now I can show that for Q of type $\mathbb{A}_n$ or Q an equioriented cycle we have an isomorphism of algebras

$$\phi : \mathbb{Z}\mathcal{CM}(Q) \to \mathcal{C}_0(Q)$$

given by

$$\mathcal{A} \mapsto \sum_{[X] \in \mathcal{A}} [X].$$

This generalises work of Reineke in [1] and [2].

References

[1] Markus Reineke. Generic extensions and multiplicative bases of quantum groups at $q = 0$. *Represent. Theory*, 5:147–163 (electronic), 2001.

[2] Markus Reineke. The monoid of families of quiver representations. *Proc. London Math. Soc. (3)*, 84(3):663–685, 2002.

135 Changchang Xi

Beijing Normal University, Beijing, China

Finitistic dimensions of Artin algebras

The finitistic dimension conjecture says that ***every Artin algebra has finite finitistic dimension.*** The conjecture was first proposed in 1960, and remains open today. To understand the conjecture, we use a new approach, namely we show that, for finite-dimensional algebras over a field, the finitistic dimension conjecture is equivalent to the following statement:

$(*)$ ***If*** B ***is a subalgebra of an algebra*** A ***such that rad***(B) ***is a left ideal in*** A***, and if*** A ***has finite finitistic dimension, then*** B ***has finite finitistic dimension.***

Thus it is of interest to consider a pair $B \subseteq A$ of algebras A and B, and to try bounding the finitistic dimension of the smaller algebra B by that of the bigger algebra A.

In this talk we shall report some recent developments on this conjecture made at BNU. In particular, we show that $(*)$ holds true for Artin algebras $B \subseteq A$ with A a semisimple extension of B. This includes the case rad(B)=rad(A)). We should notice that even under the condition rad(B)=rad(A), the module category of B is much more complicated than that of A. The contents of my talk are taken from some of my works, and a joint work with D.M.Xu on the finitistic dimensions of Artin algebras.

136 Fan Xu

Tsinghua University, Beijing, China

Toën's formula and Peng-Xiao's theorem

We give the construction of Hall algebras associated to triangulated categories with some condition of finiteness. The corresponding Hall number is supplied by Toën's formula. Using the method in the above construction, we can reprove a theorem of Peng-Xiao involving the global realization of Lie algebras associated to 2-period triangulated categories.

137 Shih-Wei Yang

Northeastern University, Boston, USA

Cluster algebras of finite type via principal minors

(joint with: *Andrei Zelevinsky*)

In a joint work in progress with Andrei Zelevinsky, there is given an explicit unified geometric realization for the cluster algebra of an arbitrary finite type having principal coefficients at some acyclic cluster. The cluster algebra in question is realized as the coordinate ring of a certain reduced double Bruhat cell in the complex semisimple algebraic group of the same Cartan-Killing type. In this realization the set of cluster variables is some explicitly determined collection of (generalized) principal minors.

138 Dan Zacharia

Syracuse University, Syracuse, USA

Auslander-Reiten sequences for modules with constant Betti numbers

(joint with: *Ed Green*)

Recall that if R is an artin algebra and M is an R-module, then its i-th Betti number $\beta_i(M)$ equals the number of indecomposable summands of P^i where

$$\cdots \to P^2 \to P^1 \to P^0 \to M \to 0$$

is a minimal projective R-resolution of M. We will consider in this talk indecomposable modules over a selfinjective artin algebra whose Betti numbers are eventually constant and study the Auslander-Reiten sequences ending at them. Among other results, we show that if M is a module with Betti numbers eventually equal to some positive integer b, then $\alpha(M) \leq b$ if each simple module has complexity larger than one. Moreover, if $b = 1$, then M lies in a component of the type $\mathbb{Z}A_\infty$ or a homogeneous tube.

139 Pu Zhang

Shanghai Jiao Tong University, Shanghai, China

Calabi-Yau objects in triangulated categories

(joint with: *Claude Cibils*)

We introduce the Calabi-Yau objects in a Hom-finite Krull-Schmidt triangulated k-category, and notice that the structure of the minimal, consequently all the CY objects, can be described. The relation between indecomposable CY objects and Auslander-Reiten triangles is provided. Finally we classify all the CY modules of self-injective Nakayama algebras, determining this way the self-injective Nakayama algebras admitting indecomposable CY modules.

140 Guodong Zhou

Université de Picardie, Amiens, France

Gentle algebras and blocks of dihedral defect

J. Schröer and A. Zimmermann proved that the stable endomorphism algebra of a module without self-extensions over a special biserial algebra is a gentle algebra. It is an interesting question to see which gentle algebras arise as stable endomorphism algebras of modules without self-extensions over blocs of dihedral defect defined over an algebrically closed field of caracteristic 2, whose socle factors are special biserial algebras, according to the classification of these blocs due to K. Erdmann and T. Holm. In my thesis, we settle this question and we obtain 65 indecomposable gentle algebras.

141 Bin Zhu

Tsinghua University, Beijing, China

$d-$cluster tiltings in $d-$cluster categories

$d-$cluster categories as a generalization of cluster categories, were introduced by Keller, Thomas, for $d \in \mathbf{N}$. we prove some properties of $d-$cluster tilting objects in these categories, including that any basic $d-$cluster tilting object contains exactly n indecomposable direct summands.

142 Anatoliy Zhuchok

Luhansk Taras Shevchenko National Pedagogical University, Luhansk, Ukraine

Automorphisms group of free product of π-regular semigroups

In this work it is proved that automorphisms group of free product of π-regular semigroups is isomorphic to the direct product of wreath products of groups.

The terminology and designations correspond to adopted ones in [1].

Let $\{S_i\}_{i\in Y}$ be a family of pairwise disjoint semigroups, Fr is the set all finite sequences $a_1a_2...a_k$ such that if $a_j \in S_{i_j}$ then $i_j \neq i_{j+1}, 1 \leq j \leq k-1$. On the set Fr the product is defined by

$$a_1a_2...a_k \bullet b_1b_2...b_s =$$
$$= \begin{cases} a_1a_2...a_kb_1b_2...b_s, & \text{if} \quad a_k \in S_i, b_1 \in S_j, i \neq j, \\ a_1a_2...a_{k-1}(a_k \cdot b_1)b_2...b_s, & \text{if } a_k,\ b_1 \in S_i, \cdot - \text{operation in } S_i,\ \ i \in Y. \end{cases}$$

for all $a_1a_2...a_k, b_1b_2...b_s \in Fr$. Set Fr concerning this operation is a semigroup. This semigroup is called the free product of semigroups S_i $(i \in Y)$ and it is denoted by $Fr[S_i]_{i\in Y}$.

Let G be an arbitrary group, X is the nonempty arbitrary set, $\Im[X]$ is symmetric group on a set X. Through $\overline{G} = \prod_{x\in X} G_x$ we denote decart product of isomorphic copies G_x of group G indexed of elements of set X and assuming

$$\rho : \Im[X] \to Aut\overline{G} : \gamma \mapsto \gamma\rho = \rho_\gamma,$$

where $\rho_\gamma((a_x)) = (a_{\gamma(x)})$ for all $(a_x) \in \overline{G}$. On the set $\overline{G} \times \Im[X]$ the product is defined by $((a_x), \gamma_1)((b_x), \gamma_2) = ((a_x)\rho_{\gamma_1}((b_x)), \gamma_1\gamma_2)$ for all $((a_x), \gamma_1), ((b_x), \gamma_2) \in \overline{G} \times \Im[X]$. The group obtained is called the wreath product of group G with symmetric group $\Im[X]$. The wreath product of group G with symmetric group $\Im[X]$ is denoted by $G\ \bar{\imath}\ \Im[X]$.

Semigroup T is called π-regular if for any $g \in T$ there exists natural k such that g^k - regular element.

Let $\{S_i\}_{i\in I}$ be a family of arbitrary pairwise disjoint π-regular semigroups $S_i, i \in I$. If $Y \subseteq I$ and $\{S_i\}_{i\in Y}$ is a family of pairwise not isomorphic semigroups of family $\{S_i\}_{i\in I}$ then through T_j we denote the set all of semigroups $S_i, i \in I$ which are isomorphic semigroup $S_j, j \in Y$. For every $j \in Y$ assuming $T_j = \{S_i\}_{i\in A_j}$ where $A_j \subseteq I$.

Theorem. Automorphisms group $AutFr[S_i]_{i\in I}$ of free product of π-regular semigroups $S_i, i \in I$ is isomorphic to the direct product $\prod_{j\in Y} AutS_j\ \bar{\imath}\ \Im[A_j]$ of wreath products of automorphisms groups $AutS_j$ of semigroups S_j with symmetric groups $\Im[A_j]$ on a sets $A_j, j \in Y$.

References

1. A.H. Klifford, G.B. Preston, The algebraic theory of semigroups, vol.1,2, American Mathematical Society, 1964,1967.

143 Yuriy Zhuchok

Luhansk Taras Shevchenko National Pedagogical University, Luhansk, Ukraine

Relational representations of quasi-ordered n-commutative semigroups

The representations of ordered semigroups by different binary relations types have been studied by many authors (see e.g. [1,2]). Here some relational representations of n-commutative quasi-ordered semigroups are described.

Recall that quasi-order on a set X is called the binary relation which is both reflexive and transitive.

Binary relation ρ on a semigroup S is called stable on the left (on the right) if for all $x, y, z \in S$ provided $x\rho y$ then it follows $zx\rho zy$ $(xz\rho yz)$. If quasi-order on a semigroup S is stable both on the left and on the right it is called as a stable one.

Semigroup S is called the quasi-ordered, if it defined some stable quasi-order in it. Quasi-ordered semigroup S with relation $\trianglelefteq$ will be denoted by $(S, \trianglelefteq)$.

Let B_X be a semigroup of all binary relations on a set X. This semigroup is ordered by relation of set-theoretical including of binary relations.

Homomorphism of semigroup S into semigroup B_X will be called relational representation of semigroup S over the set X.

If S is an arbitrary semigroup, will denoted by S^1 a semigroup S with external joined unity 1. Let n be a fixed natural number.

Binary relation ρ on the semigroup S will be called n-positive on the left (on the right) if $x^n \rho (xy)^n$ (resp. $x^n \rho (yx)^n$) for all $x, y \in S$. If $\rho \subseteq S \times S$ is both positive on the left and positive on the right then him called n-positive.

Semigroup S is called n-commutative if $(xy)^n = x^n y^n$ for all $x, y \in S$.

Let $(S, \trianglelefteq)$ be a n-commutative quasi-ordered on the left semigroup and let

$$\trianglelefteq_1 = \trianglelefteq \cup \{(1; x) \mid x \in S^1\}.$$

The map $g^{(n)}$ of semigroup $(S, \trianglelefteq)$ into semigroup B_{S^1} can be defined in that way: $x \mapsto xg^{(n)} = g_x^{(n)}$, where

$$g_x^{(n)} = \{(a; b) \in S^1 \times S^1 | a^n \trianglelefteq_1 (xb)^n\}.$$

Theorem 1. *The map $g^{(n)}$ is a relational representation of semigroup $(S, \trianglelefteq)$ over the set S^1 if and only if when quasi-order $\trianglelefteq$ is a n-positive on the left.*

We will notice that if $\trianglelefteq_1 = \trianglelefteq \cup \{(1; 1)\}$ then $g^{(n)}$ is a relational representation for each n-commutative quasi-ordered on the left semigroup $(S, \trianglelefteq)$ ([3]).

The n-symmetrization of binary relation ρ on the semigroup S is called the relation ρ_n^* on S which is defined according the rule:

$$a\rho_n^* b \Leftrightarrow (a^n \rho b^n \ \ \& \ \ b^n \rho a^n).$$

Farther, let $(S, \trianglelefteq)$ be a n-commutative quasi-ordered semigroup with a n-positive on the left relation $\trianglelefteq$ and let $\bigtriangledown_{g^{(n)}}$ be a congruence of semigroup S corresponding to the representation $g^{(n)}$.

Theorem 2. *The congruence $\bigtriangledown_{g^{(n)}}$ of relational representation $g^{(n)}$ coincides with symmetrization $\trianglelefteq_n^*$ of quasi-order relation $\trianglelefteq$ on a semigroup S.*

Except for it different properties such representations are investigated.

References:

1. Vagner V.V. *Representations of ordered semigroups* // Mathematical collection. - 1956. - V. 38 (80). - P. 203-240. (In Russian)

2. Zaretskiy K.A. *Representations of ordered semigroups by binary relations* // News of higher educational establishments. - Math. - 1959. - V. 6 (13). - P. 48-50. (In Russian)

3. Zhuchok Y.V. *Relational representations of n-commutative semigroups* // XI International scientific Kravchuk conference. - Kyiv: 2006. - P. 423. (In Russian)

144 Alexander Zimmermann

Université de Picardie, Amiens, France

Morita invariance of invariant bilinear forms on modules over group rings

(joint with: *Wolfgang Willems and Intan Muchtadi-Alamsyah*)

Let k be a field of finite characteristic $p > 0$ and let G be a finite group. If a simple kG-module V admits a non degenerate bilinear form which is invariant under the group action then the form is unique up to an automorphism of V. We study the behaviour of the existence and the type of such a form under Morita equivalences. The methods and results for odd and for even characteristic are very different. The characteristic 2 case uses a theory of polynomial endo-functors of kG-modules.

145 Gordana Todorov

Northeastern University, Boston, USA

Cluster complexes via semi-invariants

(joint with: *Kiyoshi Igusa, Kent Orr and Jerzy Weyman*)

We define and study virtual representation spaces having both positive and negative dimensions at the vertices of a quiver without oriented cycles. We consider the natural semi-invariants on these spaces which we call virtual semi-invariants, and prove that they satisfy the three basic theorems: the First Fundamental Theorem, the Saturation Theorem and the Canonical Decomposition Theorem.

In the special case of Dynkin quivers with n vertices this gives the fundamental interrelationship between supports of the semi-invariants and the Tilting Triangulation of the $(n-1)$ sphere.